Project INSPIRE Grade 2-5 Course 2 Lesson 3 (26:26)

SPEAKER: Welcome to Grades 2 to 5: Nemeth Code Symbols for Fractions and Spatial Problems, Instructional Tools, Materials, and Technology. This is "Lesson 3: Long Division." Let's look at the objectives on slide 2. You're going to be able to read and write division problems that do not have a quotient, read and write division problems with and without remainders. You're going to be able to set up a math page that has numbered division problems. And you're going to be able to read and write division problems with decimals. And you're going to be able to do that using more than one method to format the problem.

Slide 3 is the anatomy of a division problem. And I know you know this, but let's just go over our terms that we want to make sure you use with your students. My division problem is 215 divided by 3, my quotient is 71, and my remainder is 2. So the dividend is the number that's being divided, in this case, 215. The divisor is doing the work. That's 3. So it's the number that we use to divide the dividend.

My quotient is the answer, 71. And in this case, I have 2 left over. And we call that the remainder. You want your student to get these terms down and to use the same terms that that classroom teacher is using.

Let's take a look at slide 4. We have division problems that have a curved division sign. This is the most common way you'll see a division sign. And to represent that in braille, we're using dots 1-3-5. When I have a division problem without a quotient, I'm going to actually write these in linear format. So this would be actually where our student needs to do the work. So we don't give them the answer.

So for example, I have 3 into 15. 3 is my divisor. 15's my dividend. To braille this in spatial format: numeric indicator, 3, my division sign, dots 1-3-5, and my dividend, 15. Next example, 6 into 24. So I've got numeric indicator, 6, dots 1-3-5 for my division sign, and the 24. And my last one is 200 divided by 12. So same pattern: numeric indicator, 12, division sign, dots 1-3-5, 200. So when I have a math problem that's division with no quotient, no answer, treat it as a linear problem.

Let's go on to slide 5, and now talk about when we actually add in the quotient. We're moving into spatial arrangements for division problems. And as we talked about in Lesson 2, with a spatial arrangement, I'm going to leave a line, blank line, above and a blank line below, and at least one cell to the left and one cell to the right. Many of our students prefer two cells. My separation line is dots 2-5, just as it was with our addition, subtraction, and multiplication spatial problems.

In spatial division, my division separation line I'm going to use in my division problems starts above that division sign, and it goes one cell to the right of the widest part of the problem. So that's going to include any remainder. In a spatial division problem, just as in addition,

subtraction, and multiplication, there are no numeric indicators. So let's look at my sample problem here on slide 5.

180 divided by 15, with a quotient or answer of 12. Let's take a look at how I set this up. I have to plan ahead. And this is a little tricky with the Perkins braille writer. But I'm going to go ahead and talk through it by first doing the problem. So that's 15, division sign, 180. Notice, no numeric indicator. Now, right above that division sign, I have five cells of dots 2-5, my separation line. Because it starts at the division sign, it goes across, past the 180 by one cell, and then my quotient is above that separation line. And notice how my 12 lines up with the 180.

It's really important that you plan your division problem before you braille it, so that you can get everything lined up for your student. I want you to take a second and make sure you understand how this problem is formatted, before you go on to slide 6.

Slide 6. Most of the time, our division problems that we want our student to do or to check are numbered. So you're going to place the problem number on the same line as the dividend, not on the same line as the answer, but on the same line as the dividend. Remember, you need to leave one or two spaces after the end of the separation line and before whatever comes next.

So when I go to plan out, my problems if you look at how I've numbered problem 1 and problem 2, problem 1, on the line with the dividend, is 100 divided by 10. I've got my separation a line above. And then I've left, looks like, two spaces before I do my numeric indicator for the problem, 2. So this, again, takes out some time for planning.

So numeric indicator, 1, punctuation indicator period, 10, division sign, 100. My dividend's 100. I've got my division sign. I've got my divisor. I'm good. Now I need to put my quotient in. So above my division sign, I do my five cells in this case, of dots 2-5 for my separation line. And then my answer or my quotient, the 10, is above that.

So problem 2, problem 3, problem 4 are formulated very much the same way as problem 1. If you need to, go ahead and pause. Take a look at these problems. Notice, I have a blank line in both, above and below the problems, two cells between the end of the separation line and the start of the numeric indicator, for problems 2 and 4. You may even want to go ahead and braille these problems and compare to what I've done.

Alright, slide 7. We're getting tougher, gang. This is long division. This is when the teacher wants us to show our work. And for our braille readers, they really need to be able to read these problems. Asking a student to do this type of problem on a braille writer does take some work. Please do not penalize a student if they don't line up their separation lines. Some of your students may be using an abacus. And so you and the teacher and the student need to agree that the student is doing the computation in a different format but still getting the correct answer, that the abacus is not doing the work for the student.

So it's going to be important that you, that general classroom teacher, and your student have an understanding. So let's just look at how I have formatted my division problem. I have 2,955 divided by 3, and my quotient is 985. When I look at the braille, I have brailled 3, division sign,

2,955. Above that, I have a separation line that is six cells in length. So all my separation lines in this problem are going to be six cells in length. I have lined up the 985, my quotient, so that my place value is accurate.

I work through the problem. I basically have done 29 divided by 3 is 9. 9 times 3 is 27. Notice how I have lined up the 29 and the 27. My separation line, 29 minus 27 is 2. I have brought down my 5. 3 into 25 is 8. 8 times 3 is 24. I've done my subtraction. I got 1. I brought down my 5. 3 times 5 is 15. I've done my subtraction, and I get 0.

Your student needs to be able to read a problem like this to be able to understand how I've lined up my place value, you know, keeping my columns straight. Asking a student in fourth or fifth grade to do this on a braille writer for a series of problems is not realistic. The math teacher may want the student to show their work on one or two problems, to demonstrate they understand. So there's going to be some negotiation with that general ed teacher.

Let's go on to slide 8. When we talk about that showing your work and those problems are numbered, I wanted to point out, again, that our numbers go on the same line as the divisor and the dividend. That we need to make sure we're leaving our windows, so a blank line above and a blank line below. That from the end of the separation line, I want to leave one or two cells before I do my numeric indicator. So let's look at problem 27 together, and you'll see that I followed the same rules as I did in the last division problem.

So I numbered my problem, numeric indicator, 27, punctuation indicator, period. My divisor is 3. My division sign, in this case, my dividend is 51. I have my quotient is 17. So my quotient lines up above the 51. My separation line goes one cell to the right of the quotient. So in this case, my separation line is four cells. So if we work the problem, 3 into 5 goes 1 time. So I put the 1 above the 5. 1 times 3 is 3. Do my subtraction. 5 minus 3 is 2. I bring down my 1. I say, "How many times does 3 go into 21?" 7. We'll put that 7 right above the 1 in 51. 7 times 3 is 21. And my answer is 0.

And as you can see, on problem 28, snuck in that same problem, didn't I? 3 into 2,955, and I wanted to show you, I really am going to have to consider that problem 28 takes more lines to braille than problem 27. So when I go to set up my page, I want to think about where I'm going to put problem 29. It's going to be one blank line after the end of problem 28. So keep that in mind. So if you need to review problem 28 again, go for it.

Alright, slide 9. It's time for you to do work. And I put different problems in here this time. So Activity 3A. I'd like you to interline these six problems. When you're ready, come on back.

Slide 10 has the answer key for Activity 3A. So make sure that you interlined your problems properly, that you use the proper numbers, that you lined things up in problems 3 and 4 with those quotients.

Let's go on to slide 11. So did you draw your separation lines in the correct place? Did you go ahead and line all your numbers up in both problems?

Alright, let's go on to slide 12. And this time, I actually want you to do some transcribing. So we've got Activity 3B. And I've got 5 problems for you to transcribe. So go ahead and do that for me, please.

Slide 13. Let's have you check your work. So again, my first two problems do not have quotients. So you should have used your numeric indicator when you were transcribing. Problems 3 and 4 do you have quotients, so no numeric indicators. Make sure all your numbers are properly aligned, and that your separation line goes one cell to the right of the quotient.

On slide 14, we have problem 5 for you. So go ahead and check to make sure that you've brailled everything properly in Activity 3B.

Slide 15 gets into remainders. Now, when a quotient includes a remainder, you're going to put a space, followed by the letter "r" for remainder, very catchy, followed by a dot 5. Folks, this is not a dot 1. You and your students are going to want to sometimes pretend it's a dot 1. But it's a dot 5. That's our multi-purpose indicator.

In this case, the purpose is to say, "Hey braille reader, we've got a remainder." So it's "r," dot 5 to show a remainder. My separation line always extends to include my remainder, and then one cell to the right of where my remainder ends. So my example problem, that is 439 divided by 6 with a quotient of 73 remainder 1. This has a quotient, so I know, no numeric indicator. So I'm going to do my divisor, 6, my division sign, 1-3-5, and then I've got my dividend, 439, I have the 73 aligned above the tens and the ones column, space, remainder with the "r," dot 5, 1. Now, I want you to look at how I did my separation line. Do you see that it starts above the division sign, and it goes one cell to the right of the 1?

My second problem, I've shown you a long division problem of 4,529 divided by 25 with my quotient of 181, remainder 4. So as we work through this problem, we can see that I do 25 into 45 is 1, so I'm going to put that 1 above the 5 in 4,529. And then I'm going to do my math. 1 times 25 is 25. I'm going to go ahead and bring down the 2. So now I'm doing 25 into 202. That's 8. 8 times 25 is 200.

Notice how all my columns are lined up. Now, I've got 25 into 29. That goes one time, remainder 4. The important thing is, I want to figure out how long my separation line is going to be from the get-go. That's going to be ten cells. My separation line starts above the division sign. It goes one cell to the right of the remainder, consistently, all the way through.

Slide 16. I want to show you an alternative way for doing division, especially if our students need to show their work. Because trying to line up and down, and figure out, do I need an extended separation line, because I'm going to have a remainder, we know that's not realistic. So how can we support our braille readers in doing division with their Perkins braille writer? Now, your student doesn't need to know all their multiplication facts to be able to use this method. Because I'm going to be able to see what's left over and work my way through.

This is a method that general ed teachers may be calling the "t-method" or the "hangman method." So this is something that students are going to hear about in class. So let's see how this

works. So in this particular case, my division problem is 78 divided by 6. So going to to put that separation line above. And I'm going to encourage my student to do several cells beyond the problem.

Then my student is going to be using dots 4-5-6 to divide the left and the right side of the problem. So I brailled the problem 6, division sign, 78. I put a space. I put a 4-5-6. five Now, my student thinks, Okay, how many 6s are in 78? Well, I know for sure there's going to be at least 5. So my student goes ahead and brailles a 5. In this case I would have my student leave two spaces to braille the 5.

OK, 6 times 5 is 30. So my student brailles a 30, brailles a separation line, dots 4-5-6 to add separation between where I'm brailling my quotient part and the problem, and does the subtraction. So 78 minus 30 is 48. My student thinks, how many 6s are in 48? Well, I know for sure, at least 4.

So the student brailles the 4, does the math. 4 times 6 is 24. So my student has that 24 lined up right underneath the 48, does a separation line over again to the dots 4-5-6, does the subtraction. 48 minus 24 is 24. Oh, well hey, I know I need another 4 there. So 4 times 6 is 24. And then I get an answer of 0.

Now, my student on the right-hand side needs to line up and add the 5, the 4, and the 4 to get their actual quotient. So 5 and 4 is 9, and 4 more is 13. So my quotient is 13. In this method, my student doesn't have to know exactly every multiplication fact. My student doesn't have to roll the paper in the braille writer. And in the end, my student gets their quotient by adding their approximations as they divide it. I like this method. I wish, as a kid, I would have learned this method. I'm sure it would have made it a lot easier.

Alright, let's go on to slide 17 and look at that same problem another way. So I've got 78 divided by 6. This particular student is like, Okay, I know that 6 times 10 is 60. So our student has brailled 6 into 78 with the division sign, the separation line above. And then after the 78, he's going to braille the space, and then the 4-5-6 to separate the side of the problem with where I'm working my quotient. The student brailled 10. So 6 times 10 is 60. The student subtracted, got 18.

Now, how many 6's are in 18? 3. So my student brailles a 3 on the quotient side. Again, that separation line is going all the way across. This is the very top separation line. My student adds 10 plus 3 and gets that same quotient of 13. Regardless of where our student is with their multiplication facts, they can work their way down to get in the proper quotient.

Slide 18. Let's talk about decimals. When we put a decimal into a problem, they're going to occur in the dividend. And you're going to leave a blank cell as you work your way down through showing your work in this long division problem. So rather than the student having to constantly braille that decimal point of dots 4 6, they're going to leave a blank cell for it.

So if I have the problem 53.75 divided by 25, with a quotient of 2.15, I'm going to start out brailling the problem: 25, division sign, 53, decimal point, 75. Need to see the decimal point

there. Okay, I'm going to leave my separation line above the problem. And then in this case, my quotient is 2.15. So I am going to put that decimal point in my quotient.

But as my student is doing their work, they don't need that decimal point. So for example, I start out with 2 times 25 is 50. My student is going to take 53 and subtract 50 and get a 3. So you're going to do 3, space, where that decimal point would be, and bring down the 7 from the 53.75. So now, there is just a space there. But my student knows that that's 37. So 1 times 25 is 25. So my student's going to put a 2 under the 3 and a 5 under the 7. Again, there's that space they're holding for the decimal point.

And do their subtraction. They're going to get 1 space 2. Bring down the 5. So 5 times 25 is 125. The student does the subtraction, gets 0. So my quotient is 2.15. As my student did the work, they left just a space where the decimal point goes. Again, your student really needs to know how to read these problems. They may be doing the problem on their abacus. They may be doing the problem on their math window. If the other students have to do, you know, 10 of these math problems and show their work, you and the student and the math teacher may negotiate that your student chose to work on two or three of them, because it does take them longer.

Let's move on to slide 19. In method 2, I've actually put the decimal place in when it occurs. And I repeat as I bring it down. For some of our students, seeing that dot 4-6 for the decimal point, rather than a space, is going to help them know where they are in the problem. So I'll go ahead and pause for a second and let you read through this problem. It's the exact same problem we just worked on slide 18. So here on slide 19, you have 53.75 divided by 25, with a quotient of 2.15. Notice how the decimal point is included this time.

Alright, let's take a look at slide 20. Now, in this instance, we have a decimal in the divisor. So let's take a look at my problem. It's 6.76 divided by 1.3. Now, because I have 1.3 in my divisor, that's a big no-no. So I'm going to use the caret, which in braille is dots 4-5-6, 1-2-6, to show that I've moved that decimal place over, in this case, to the right.

So my new problem becomes 13 into 67.6. Because if I move my decimal place over in the divisor, I've got to move it over in the dividend. Then I just go along, and I do my division. So I say how many 13s are in 67? 5. So I'm going to put a 5 in my quotient, above that separation line 5 times 13 is 65. Do my subtraction. I get 2. I'm going to leave a space for where that caret goes, and I'm going to bring down my 6. So really, I'm saying 13 into 26 is 2.

Now, notice that my quotient, I am going to leave a space, and then put my decimal point, so I remember where it actually goes when I'm doing my answer. So then I just do my multiplication. 2 times 13 is 26. And voila! I get 0. So my final answer is 5.2.

Now, in reality, you want your students to be able to read these problems. Most of our students are not going to be brailling them out with the caret. What you want your student to do is to braille that problem that they're going to do, and actually do the problem of 67.6 divided by 13, rather than actually dealing with the caret when they're brailling. And remember, they could use the abacus. They could use the math window. They could use one of the other methods, like the "t-method" that we talked about.

Let's go on to slide 21 and look at another method, when I have a decimal in the divisor. So in this method, I have that original problem, the 6.76 divided by 1.3. What I'm going to do is, I'm going to make my devisor a whole number of 13, which means I have to move my decimal point to accomplish that. So now I'm doing 67.6 divided by 13.

And I can go back to that method of leaving a space for my decimal point as I do my computation, or I can actually put the decimal point in, whatever method I prefer. So on this problem, I say how many 13s are in 67? 5! 5 times 13 is 65. I do my subtraction. I get 2. Can either leave a space or a decimal point, and bring down my 6. How many 13s are in 26? 2. 2 times 13 is 26, and I get 0. So my quotient is 5.2.

Slide 22 puts you to work. I need you to interline the four problems that you are seeing here.

Slide 23 has the answer key for Activity 3C. So we have a remainder in problem 1. So did you interline properly, and put in your "r" for remainder, but not 13, just "r"? Because that's a dot 5, our multi-purpose dot, and not the number 1.

In problem 2, did you show the caret in print?

Let's go on to slide 24 and look at the answer key for problems 3 and 4 here in Activity 3C. So check carefully to make sure that you used our alternative methods properly, that you have your numbers lined up. And problem number 4, that you used your decimal point.

So this concludes Lesson 3 on division problems. Go ahead and move on to Lesson 4, where you're going to learn some more information about formatting materials for second through fifth graders. Thank you so much.