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Web-Based Resources for Mathematics: Articles Related to Mathematics Education

Curriculum Enhancement

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National Center on Accessing the General Curriculum

IDEAs that Work
U.S. Office of Special Education Programs

INTRODUCTION

A vast number of resources for teaching mathematics in the elementary and secondary grades are now available on the Internet. Many of these resources are available for free. The resources discussed below are articles related to mathematics education. The articles have been catalogued according to the content and process standards developed by the National Council of Teachers of Mathematics (NCTM) and have further been reviewed for their type of research study. Because of the vastness of the World Wide Web and the speed with which it changes, not all resources can be included in this discussion.

ARTICLES RELATED TO MATHEMATICS EDUCATION

The articles available in this document are available for free on the Internet. Although some are detailed reports of published research, most are written for teachers and parents. While they may describe "aspects of research," and may give some examples, they do not present all of the details. Rather, the articles highlight findings or anecdotes that teachers might find relevant for individual classrooms or students. We have classified the articles in order to help distinguish the different types and their research base. Inclusion of an article does not imply endorsement of its content; it is up to the reader to decide if the information is sound and relevant.

Types of Research in Education

Gersten (2001) has presented a framework for classifying research in education. He describes two categories of research: descriptive and experimental. Descriptive research may use either qualitative or quantitative research methods. It is useful for understanding a subject and can contribute to theory development. Good descriptive studies provide a rich discussion of an area of interest, and their results often suggest interventions that then can be tested through experimental designs. Descriptive studies can help teachers have a deeper understanding of situations that occur in a classroom and may stimulate thinking on possible approaches to take. However, they do not provide validation for any teaching method or approach.

Experimental research typically studies the effects of an intervention. Studies may assess the effects of the intervention on a group of students, or compare the changes in a group or students receiving the intervention to changes in one that does not receive the intervention. Gersten (2001) divides experimental research into two types. The first includes small-scale, highly controlled studies. The number of subjects is limited, and the experimental situation is generally carefully controlled. No single study of this type is definitive; studies must be replicated in order for results to be validated. Large-scale field studies comprise the second type. Frequently, large-scale studies attempt to validate the findings of small-scale experimental research. Large numbers of students from many classrooms or schools participate in these studies. The educational environment is not as tightly controlled as in the small-scale studies. Gersten describes these studies as "field tests" of what actually works. To demonstrate that an intervention works, large-scale studies must include a control group that receives no special intervention, and students or, more typically, classrooms must be randomly assigned to either the intervention group or the control group. The students, teachers, and classroom situations should be similar for both groups at the beginning of the study.

Classification of Articles

Classification	Description	Classroom Applications	Primary Audience
Research Report: Experimental	Article describes complete experimental research study, including hypotheses or research questions, subjects, research setting, methods used (including measures), results (including statistical analysis), conclusions or discussion, and limitations.	Research reports generally include detailed information on the methods and results of the research, which may not be directly applicable to an individual teacher's students or curriculum. However, the discussion section often sites the study within the broader body of research in the area and frequently delineates implications for teaching and curriculum development.	Researchers
Research Report: Descriptive	Article describes complete descriptive research study, including hypotheses or research questions, subjects, research setting, methods used (including measures), results, conclusions or discussion, and limitations.		Researchers
Research Review	Article presents review of published research on a specific topic or topics. Reviews may include very broad types of studies. Authors may or may not evaluate the quality of these studies, so the reader needs to be aware of this.	Often these articles include conclusions that can be drawn on by teachers when developing curriculum or planning for individual students.	Researchers and Practitioners
Theoretical/ Conceptual Article	Article presents theoretical or conceptual framework on a particular topic, based in pedagogy and citing research and examples from practice that support the theory.	Teachers can apply the theory or concept when developing curriculum or planning for individual students. They need to be aware of the rigor of the research supporting the theory presented and understand that there are most likely competing theories.	Researchers and Practitioners
Suggestions Based on Research	Article presents suggestions based on published research on a specific topic or topics but does not provide comprehensive review of research. Because details of the study itself are not presented, the reader cannot judge the rigor of the research.	Teachers can evaluate these ideas within their own classroom. Because the suggestions are derived from research, they are typically more specific about when they might be effective, and they may have more potential than suggestions based solely on anecdotes.	Practitioners
Anecdotes from Research Study	Article presents anecdotes and/or observations of students and activities taken from a specific research study, but data are not reported. Because details of the study itself are not presented, the reader cannot judge the rigor of the research.	These may present ideas of what to do when a teacher encounters a situation similar to one presented in the article.	Practitioners
Anecdotes or Suggestions Based on Practice or From Experts in Field	Article is not overtly research-based but provides suggestions based on the author's knowledge and experiences.	These are similar to talking with other teachers about their experiences. The suggestions are not research-based, but may be useful in specific situations.	Practitioners

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Number and Operations

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Knowing and Doing Math Improve Mathematics Achievement. (2002, Fall). *Research Connections in Special Education No. 11.* Retrieved July 13, 2004, from http://ericec.org/osep/recon11/rc11sec2.html (**Note**: This URL is no longer active.)

McClain, K., Cobb, P., & Bowers, J. (1998). A Contextual Investigation of Three-Digit Addition and Subtraction. In L. J. Morrow & M. J. Kenney (Eds.), *The Teaching and Learning of Algorithms in School Mathematics, 1998 Yearbook* (pp. 141-150). Reston, VA: NCTM. http://www.wcer.wisc.edu/ncisla/publications/articles/AddSubtract.pdf

McClain, K., Cobb, P., Gravemeijer, K., & Estes, B. (1999). Developing Mathematical Reasoning within the Context of Measurement. In V.L. Stiff & R.F. Curcio (Eds.), *Developing Mathematical Reasoning in Grades K-12, 1999 Yearbook* (pp. 93-106). Reston, VA: NCTM. http://www.wcer.wisc.edu/ncisla/publications/articles/AddSubtract.pdf

Reys, B.J., & Reys, R.E. (1990). Estimation – Direction from the Standards. *Arithmetic Teacher, 37*(7), 22-25. http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-002218-2218,00.shtm

Algebra

Falkner, K.P., Levi, L., & Carpenter, T.P. (1999). Children's Understanding of Equality: A Foundation for Algebra. *Teaching Children Mathematics, 6*(4), 232-236. http://www.wcer.wisc.edu/ncisla/publications/articles/AlgebraNCTM.pdf

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Geometry

Battista, M.T., & Clements, D.H. (1991). Using Spatial Imagery in Geometric Reasoning. *Arithmetic Teacher, 39*(3), 18-21. http://www.terc.edu/investigations/relevant/html/UsingSpatialImgery.html

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Rowan, T.E. (n.d.). *The Geometry Standards in K-8 Mathematics*. Retrieved July 13, 2004, from http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-002184-2184,00.shtm

Measurement

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Data Analysis and Probability

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Problem-Solving

Knowing and Doing Math Improve Mathematics Achievement. (2002, Fall). *Research Connections in Special Education No. 11*. Retrieved July 13, 2004, from http://ericec.org/osep/recon11/rc11sec2.html (**Note**: This URL is no longer active.)

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Communication

Falkner, K.P., Levi, L., & Carpenter, T.P. (1999). Children’s Understanding of Equality: A Foundation for Algebra. *Teaching Children Mathematics, 6*(4), 232-236. http://www.wcer.wisc.edu/ncisla/publications/articles/AlgebraNCTM.pdf

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Connections

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Representation

Clements, D H., & McMillen, S. (1996). Rethinking “Concrete” Manipulatives. *Teaching Children Mathematics, 2*(5), 270-279. http://www.terc.edu/investigations/relevant/html/rethinkingconcrete.html

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REVIEW OF THE ARTICLES

Battista, M.T., & Clements, D.H. (1991). Using Spatial Imagery in Geometric Reasoning. *Arithmetic Teacher, 39*(3), 18-21.
http://www.terc.edu/investigations/relevant/html/UsingSpatialImgery.html

What it's about	The article uses LOGO or Turtle Graphics activities to illustrate how children use spatial imagery to understand the characteristics of geometric shapes and how their reasoning becomes more abstract and analytical over time. It gives specific examples of students using spatial imagery, and provides suggestions for fostering this skill in students.
How it addresses standards	*Geometry.* It describes how students can analyze characteristics of two-dimensional shapes and use visualization, spatial reasoning, and geometric modeling *Reasoning and Proof.* It describes different levels at which elementary students make and investigate mathematical conjectures.
How to use	The article provides specific activities that can be done with LOGO or Turtle Graphics that support students in spatial reasoning. Student discussion of the characteristics of rectangles can be generated through examples, such as the quadrilateral figures illustrated in the article.
Grade levels	• Elementary
Research	Suggestions based on research

Clements, D.H., & McMillen, S. (1996). Rethinking "Concrete" Manipulatives. *Teaching Children Mathematics, 2*(5), 270-279.
http://www.terc.edu/investigations/relevant/html/rethinkingconcrete.html

What it's about	This article summarizes research on the use of manipulatives in mathematics. It describes problems some students have in understanding what the manipulatives represent, and it provides recommendations for their use. The article also compares physical manipulatives with computer-based manipulatives.
How it addresses standard	*Representation.* The article describes appropriate ways to use manipulatives as representations to model and interpret mathematical phenomena.
How to use	The article gives recommendations for selecting and using different manipulatives. It describes how to introduce them so students understand the concepts they are representing, and highlights potential problems students may encounter in understanding their meaning.
Grade levels	• Elementary • Middle school • High school Although the article was written for elementary grades, research supports the use of manipulatives at all grades, including high school, for understanding mathematical concepts.

Research	Suggestions based on research

Falkner, K.P., Levi, L., & Carpenter, T.P. (1999). Children's Understanding of Equality: A Foundation for Algebra. *Teaching Children Mathematics, 6*(4), 232-236. http://www.wcer.wisc.edu/ncisla/publications/articles/AlgebraNCTM.pdf

What it's about	The article describes misconceptions about the meaning of the equals sign in the elementary grades. "Children...generally think that the equals sign means that they should carry out the calculation that precedes it and that the number after the equals sign is the answer to the calculation." They often do not see the equals sign as meaning "the same as." The article describes the experience of one teacher and her first and second grade students, who were part of a research study. It presents highlights of the discussions the students had throughout the year as they struggled with understanding the equals sign and the ways that the teacher supported this understanding through class discussions.
How it addresses standard	*Algebra.* It describes a significant misconception students have regarding an algebraic symbol (the equals sign). *Communication.* Through ongoing class discussions throughout the year students communicate their mathematical thinking to their peers and teacher.
How to use	The article highlights a common misconception in the elementary grades about the meaning of the equals sign and provides an example of how a teacher helped her students to broaden their understanding over time, using a variety of problems and ongoing class discussion to monitor their thinking. It is interesting that even sixth grade students made errors solving the problem 8 + 4 = __ + 5, giving 12 as the number that went in the blank.
Grade levels	• Elementary
Research	Suggestions based on research

Fuchs, L., & Fuchs, D. (2002). Mathematical Problem-Solving Profiles of Students with Mathematics Disabilities With and Without Comorbid Reading Disabilities. *Journal of Learning Disabilities, 35*(6), 563-573. http://www.ldonline.org/ld_indepth/math_skills/problem_solving_profiles.html

What it's about	This is a report of research on how students with and without math disabilities solve story problems. It describes a hierarchy of these problems: one-step arithmetic story problems involving only basic facts, complex story problems that require selecting the appropriate problem-solving strategy, and real-world problem-solving that includes a multiparagraph narrative of a realistic situation. In the latter two categories, the researchers were able to separate skill in problem-solving and skill with arithmetic operations. Students with math disabilities demonstrated more difficulty with problem-solving than with arithmetic operations.
How it addresses standard	*Problem-Solving.* It describes how students apply and adapt appropriate strategies to solve problems in different contexts, and describes difficulties they may encounter in doing this.

How to use	The article provides information on the types of difficulties research indicates are commonly seen in students with math disabilities. It also gives a detailed description of the different levels of problems that they used and an analysis of what aspects were difficult for the students who struggled with math. The description of the problems can be useful in selecting word problems and analyzing why some students are making errors on them.
Grade levels	• Elementary
Research	Research report: descriptive

Gagnon, J.C., & Maccini, P. (2001). Preparing students with disabilities for algebra. *Teaching Exceptional Children, 34*(1), 8-15. http://journals.cec.sped.org/EC/Archive_Articles/VOL.34NO.1SEPTOCT2001_TEC_gagnon.pdf

What it's about	This article reviews both the standards and principles of mathematics that have been developed by the National Council of Teachers of Mathematics (NCTM). Then, based on a review of research on successful methods of teaching algebra to secondary students with learning disabilities, the authors describe seven effective teaching strategies and provide examples for each.
How it addresses standard	*Algebra.* The article describes teaching strategies that support students in learning how to represent and analyze mathematical problems. *Representation.* The teaching methods include ones that teach students to create representations that model problems.
How to use	The article provides a concise summary of the NCTM principles and standards. The teaching strategies are described clearly, with good illustrations, and can be used with any algebra curriculum.
Grade levels	• Secondary
Research	Anecdotes or suggestions based on practice or from experts in field

Garnett, K. (1992). Developing Fluency with Basic Number Facts: Intervention for Students with Learning Disabilities. *Learning Disabilities Research & Practice, 7*, 210-216. http://www.ldonline.org/ld_indepth/math_skills/garnett_ldrp.html

What it's about	The article reviews research on mathematics disabilities, and describes research from cognitive psychology on how children develop strategies to solve basic facts and how this pattern differs in some children with learning disabilities. It describes ways to assess the strategies a student uses with addition problems, and gives suggestions for instruction based on cognitive psychology research.
How it addresses standard	*Number and Operations.* The article discusses research on how students learn to compute fluently.

How to use	The article describes strategies children typically use when learning basic addition facts. Students use a variety of these strategies at any given time. The author describes ways to design assessment and instruction based on the pattern of strategies a student uses.
Grade levels	• Elementary The information is also useful for older children who struggle with basic facts.
Research	Research review

Garnett, K. (1998). *Math Learning Disabilities*. Retrieved July 13, 2004, from http://www.ldonline.org/ld_indepth/math_skills/garnett.html

What it's about	Although the article focuses on learning disabilities in mathematics, the information is relevant for any classroom in which some children struggle with mathematics. The article points out that many students who have learning difficulties in mathematics are not referred for evaluation or provided with special services. The article describes problems that students may encounter and has suggestions for supporting these students.
How it addresses standard	*Number and Operations.* The article gives suggestions for helping students understand the meanings of basic operations and to learn to compute fluently. *Communication.* The article gives suggestions on how to support students with language difficulties in communicating mathematical thinking.
How to use	The article describes five types of students who struggle with math: those who have trouble mastering basic facts, those with a good grasp of math concepts but poor computation skills, those who find it difficult to connect an informal understanding of math with the formal systems, those who are hampered because of language issues, and those who have trouble with the visual-spatial aspects of mathematics. Citing research, the author provides a description of each problem and suggestions for working with it. Many of the suggestions can be incorporated into the math curriculum.
Grade levels	• Elementary • Middle school
Research	Research review

Geary, D.C. (2004). *Mathematical Disabilities: What We Know and Don't Know*. Retrieved July 13, 2004, from http://www.ldonline.org/ld_indepth/math_skills/geary_math_dis.html

What it's about	This article describes what research has found about students with math disabilities both with and without concomitant reading disabilities. It gives descriptions of how children learn to count and learn to perform basic arithmetic operations, and the ways in which children with mathematics disabilities differ in their learning of these skills.

How it addresses standard	*Number and Operations.* It describes how students develop an understanding of relationships among numbers and the meaning of basic operations, and how students learn to compute fluently.
How to use	The article gives an excellent description of how children typically learn basic number skills, including counting and learning basic facts. The descriptions of children who have difficulty learning these skills are helpful when looking at students who struggle with basic arithmetic.
Grade levels	• Elementary
Research	Research review

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 44*, 18-28. http://www.ldonline.org/ld_indepth/math_skills/gersten_dyscalculia.html

What it's about	The article cites research from cognitive science relating to number sense. It demonstrates that number sense is as important to mathematics learning as phonemic awareness is to reading. Research findings on number sense are used to provide a framework for teaching and supporting students who struggle with math.
How it addresses standard	*Number and Operations.* It describes how students develop an understanding of numbers and the meanings of operations.
How to use	The article gives suggestions, based on research, for supporting students in developing number sense. It explains why rote drill alone is insufficient, and explains the importance of student self-explanations and frequent teacher feedback.
Grade levels	• Elementary • Middle school
Research	Research review

Kamii, C., & Joseph, L. (1988). Teaching Place Value and Double-Column Addition. *Arithmetic Teacher, 35*(6), 48-52. http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ACQ-104912-4912,00.shtm

What it's about	This article describes research in which the authors allowed students to develop their own ways of solving two-digit addition problems and, through the students' explanations and discussions, supported a stronger understanding of place value in these problems.
How it addresses standard	*Number and Operations.* It describes how one teacher supported her students in developing an understanding of the meaning of the operation of addition with two-digit numbers and its relationship to our decimal number system. *Communication.* It describes how the teacher facilitated class discussions in which students communicated their mathematical thinking and evaluated the strategies of others.

How to use	The article describes an example of how the teacher in the study organized a lesson on two-digit addition and facilitated class discussion. The authors describe how their approach differs from traditional instruction and the rationale behind their approach. Using their approach may help teachers to understand the thinking behind student work in this area.
Grade levels	• Elementary
Research	Suggestions based on research

Knowing and Doing Math Improve Mathematics Achievement. (2002, Fall). *Research Connections No. 11*. Retrieved July 13, 2004, from http://ericec.org/osep/recon11/rc11sec2.html (**Note**: This URL is no longer active.)

What it's about	This paper provides a brief summary of research in mathematics that was supported by the Office of Special Education Programs of the U. S. Department of Education. It includes the approaches the researchers used in the classrooms and descriptions of teachers' experiences in implementing them. Two of the studies focused on aspects of number and operations (basic principles of subtraction and flexible understanding of numbers), and one on problem-solving. Although the research was aimed at special education students, the approaches used are appropriate for any mathematics curriculum.
How it addresses standard	*Number and Operations.* It describes different ways to support students' understanding of relationships among numbers and the meanings of operations. *Problem-Solving.* Monitoring, and reflecting on, the process of problem-solving is incorporated.
How to use	The researchers cited here have a long history of conducting research on mathematics education. They each provide examples that suggest ways of approaching instruction. In addition, a list of the basic principles for doing subtraction and a strategy for arithmetic problem-solving are included. *NOTE: The link that is provided for curriculum materials for flexible understanding of numbers is incorrect. These can now be found at www.ups.edu/faculty/woodward/downloads.htm.*
Grade levels	• Elementary • Middle school
Research	Research review

Lehrer, R., & Curtis, C.L. (2000). Why Are Some Solids Perfect? Conjectures and Experiments by Third Graders. *Teaching Children Mathematics, 6*(5), 324-329. http://www.wcer.wisc.edu/ncisla/publications/articles/SolidsNCTM.pdf

What it's about	This article describes an activity in a third grade classroom in which students used Polydrons (interlocking plastic polygonal figures) to explore and eventually define the properties of Platonic, or "perfect" solids (regular solids in which all sides are equal, all angles are the same, and all faces are identical).

How it addresses standard	*Geometry.* The students analyze characteristics and properties of three-dimensional geometric shapes. *Reasoning and Proof.* The activity was used as an avenue for making and investigating mathematical conjectures, then proving or disproving them. *Communication.* Students communicated their mathematical thinking, and analyzed and evaluated the mathematical thinking of their classmates.
How to use	The article presents the background experiences that support this investigation. It describes the reasoning process of the students and the information and support provided by the teacher. The authors state that this process of conjecture and experimentation can be applied to many different aspects of mathematics.
Grade levels	• Elementary • Middle school The actual activity was conducted with third graders, but the process is appropriate for any level, particularly elementary and middle school.
Research	Anecdotes or suggestions based on practice or from experts in field

McClain, K., Cobb, P., & Bowers, J. (1998). A Contextual Investigation of Three-Digit Addition and Subtraction. In L.J. Morrow & M.J. Kenney (Eds.), *The Teaching and Learning of Algorithms in School Mathematics, 1998 Yearbook* (pp. 141-150). Reston, VA: NCTM. http://www.wcer.wisc.edu/ncisla/publications/articles/AddSubtract.pdf

What it's about	The article describes episodes that took place during a nine-week research study in which third grade students explored place value and three-digit computation within the context of a candy factory. During the instructional sequence, the students developed and shared their processes for three-digit addition and subtraction. Through the process the students became increasingly sophisticated in their numerical reasoning.
How it addresses standard	*Number and Operations.* Through the instructional activity, students developed an understanding of the meanings of the operations (addition and subtraction). *Communication.* Through class discussions, students communicated their mathematical thinking to peers and teachers. *Representation.* The students created and used representations (of the candy inventories) to organize and record their understanding of three-digit addition and subtraction.
How to use	The activity presented can be carried out in any classroom, and provides a real-world context for exploring addition and subtraction, grounding them in an understanding of place value. In addition to showing several examples of student work, the article gives highlights of the ways in which the teacher involved the students in discussion and the students' rationales for their solutions to problems and rules that they developed.
Grade levels	• Elementary The specific activity is described for third grade but could be modified for other elementary grades.

Research	Anecdotes from research study

McClain, K., Cobb, P., Gravemeijer, K., & Estes, B. (1999). Developing Mathematical Reasoning within the Context of Measurement. In V.L. Stiff & R.F. Curcio (Eds.), *Developing Mathematical Reasoning in Grades K-12, 1999 Yearbook* (pp. 93-106). Reston, VA: NCTM. http://www.wcer.wisc.edu/ncisla/publications/articles/AddSubtract.pdf

What it's about	The article describes episodes in a first grade classroom that were part of a four-month research study. The episodes highlight how the students developed meaningful understandings of measurement and used reasoning to interpret their solutions to problems and to develop estimation strategies.
How it addresses standard	*Number and Operations.* The students developed strategies for making reasonable estimates. *Measurement.* The students learned the measurable attributes of objects and processes of measurement. *Reasoning and Proof.* The students made and investigated mathematical conjectures. *Communication.* The students communicated their own mathematical thinking and analyzed the strategies of others.
How to use	The article gives a sequence of activities that supports understanding basic concepts of measurement, which then can be used to support the development of estimation strategies. The examples of student reasoning highlight the different ways that students approach these problems, some of their errors in thinking, and how the teacher (and classmates) helped guide the thinking of the students who made errors.
Grade levels	• Elementary The study was conducted on first grade students, but the reasoning examples are appropriate for early elementary.
Research	Anecdotes from research study

McClain, K., McGatha, M., & Hodge, L. (2000). Improving Data Analysis Through Discourse. *Mathematics Teaching in the Middle School, 5*(8), 548-553. http://www.wcer.wisc.edu/ncisla/publications/articles/StatisticsNCTM2.pdf

What it's about	The article describes an instructional sequence from a research study designed to help middle school students develop statistical reasoning. Students were given a data set, and then shared the ways they analyzed the data.
How it addresses standard	*Data Analysis and Probability.* Students selected and used appropriate statistical methods to analyze their data. They developed and evaluated inferences based on the data. *Reasoning and Proof.* Students developed and evaluated mathematical arguments using statistical displays. *Communication.* Students communicated their own mathematical thinking and evaluated the strategies of others.
How to use	The task given to students (analyzing the data from two SAT preparation courses to see which is more effective) provides a good example of an open-ended question. The article shows the different ways in which students approached this analysis and how the teacher guided the discussion.

Grade levels	• Middle school
Research	Anecdotes from research study

National Center for Improving Student Learning & Achievement in Mathematics & Science. (2000). *Building a Foundation for Learning Algebra in the Elementary Grades* (Vol. 1). Madison, WI: Author. http://www.wcer.wisc.edu/ncisla/publications/briefs/fall2000.pdf

What it's about	The article reviews the early algebra research project conducted in the Madison Public Schools. It describes what was done to support algebraic reasoning in first through fifth grades.
How it addresses standard	*Algebra.* The project is designed to help students make explicit and understand the underlying patterns, relations, and functions in arithmetic. *Reasoning and Proof.* Through the course of the project, students develop and evaluate mathematical arguments related to the structure of arithmetic.
How to use	The article presents suggestions for building a foundation in learning algebra, and presents illustrations of problems and class discussions that occurred.
Grade levels	• Elementary
Research	Anecdotes from research study

National Center for Improving Student Learning in Mathematics and Science. (2004). *Designing Statistics Instruction for Middle School Children* (Vol. 4). Madison, WI: Author. http://www.wcer.wisc.edu/ncisla/publications/briefs/InBrief01_04.pdf

What it's about	This is a report from a two-year project with middle school students and their teachers. The project incorporated carefully designed instruction and professional development to provide students with an understanding of statistics.
How it addresses standard	*Data Analysis and Probability.* As part of the instruction, students learned to formulate appropriate questions, select appropriate statistical methods, and make inferences and predictions based on data. *Communication.* An integral part of the instructional program was communication of students' mathematical thinking, both orally and in written form. Through whole-class discussions, students learned to evaluate the mathematical thinking of their classmates.
How to use	The article describes a sequence of instruction, moving from informal, personally relevant activities to more formal analysis of data. It emphasizes the importance of conducting data analysis on problems that students consider realistic and legitimate. Students are encouraged to use different approaches to analysis and to share their thinking with each other through discussion and written reports. The study used computer tools available with the NCTM publication *Navigating through Data Analysis in Grades 6-8.*
Grade levels	• Middle school
Research	Suggestions based on research

Penner, E., & Lehrer, R. (2000). The Shape of Fairness. *Teaching Children Mathematics, 7*(4), 210-214. http://www.wcer.wisc.edu/ncisla/publications/articles/FairnessNCTMgraphics.pdf

What it's about	Students play the game "Mother, May I" and try to determine how to position the players so they are all the same distance from "mother." Students create representations of "fair" arrangements and test them out. They discuss the relative merits of different shapes and try them out both with their representations and on the playground, making the connection between the representation and the actual situation.
How it addresses standard	*Geometry.* Students discuss the attributes of various shapes. *Communication.* Students communicate their mathematical thinking as they try different representations. *Representation.* Students use representations to model and interpret physical phenomena.
How to use	The article shows how to take a familiar game and, through modeling and discussion, have students explore representation. Students move from using models with actual people to using shapes to represent the configurations. It also demonstrates an activity in which students may come up with more than one solution.
Grade levels	• Elementary
Research	Anecdotes or suggestions based on practice or from experts in field

Powell, A. (n.d.). *Creating a Caring Community in Math Class*. Columbus, OH: Eisenhower National Clearinghouse. http://www.enc.org/features/focus/archive/ideashsmath/document.shtm?input=FOC-003510-index

What it's about	The article describes how one high school mathematics teacher engages students in entry-level math classes (classes for students who are not college-bound). She describes teachers with a caring ethic as being available to students, being responsive to their questions, and providing high-quality explanations. The article gives examples of ways in which to make mathematics relevant to students and to engage them in successful problem-solving.
How it addresses standard	*Problem-Solving.* It describes ways to help students build their mathematical knowledge through solving problems. *Connections.* By relating mathematics to students' daily lives and interests, students recognize and apply mathematics in contexts outside of mathematics.
How to use	The article includes sample autobiographical questions which can be used to make mathematics more relevant to the students by connecting it to their daily lives. It also describes the types of problems that the author has found effective in engaging struggling students and building their self-confidence in mathematics. Resources for interesting problems to use in the classroom are included at the end.
Grade levels	• High school The idea presented here could apply to any grade.
Research	Anecdotes or suggestions based on practice or from experts in field

Knowing and Doing Math Improve Mathematics Achievement. (2002, Fall). *Research Connections in Special Education, No. 11*. Retrieved July 13, 2004, from http://ericec.org/osep/recon11/rc11sec2.html (**Note**: This URL is no longer active.)

What it's about	This paper provides a brief summary of research in mathematics that was supported by the Office of Special Education Programs of the U. S. Department of Education. It includes the approaches the researchers used in the classrooms and descriptions of teachers' experiences in implementing them. Two of the studies focused on aspects of Number and Operations (Basic Principles of Subtraction and Flexible Understanding of Numbers), and one on Problem-Solving. Although the research was aimed at special education students, the approaches used are appropriate for any mathematics curriculum.
How it addresses standard	*Number and Operations.* It describes different ways to support students' understanding of relationships among numbers and the meanings of operations. *Problem-Solving.* Monitoring and reflecting on the process of problem-solving is incorporated.
How to use	The researchers cited here have a long history of conducting research on mathematics education. They each provide examples that suggest ways of approaching instruction. In addition, a list of the basic principles for doing subtraction and a strategy for arithmetic problem-solving are included. *NOTE: The link that is provided for curriculum materials for flexible understanding of numbers is incorrect. These can now be found at www.ups.edu/faculty/woodward/downloads.htm.*
Grade levels	• Elementary • Middle school
Research	Research review

Reys, B.J., & Reys, R.E. (1990). Estimation – Direction from the Standards. *Arithmetic Teacher, 37*(7), 22-25. http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-002218-2218,00.shtm

What it's about	This article takes a broad perspective on estimation and its importance, both in the mathematics curriculum and in everyday life. The authors describe estimation as going beyond the traditionally-taught rounding rules and integrating it into all aspects of number concepts.
How it addresses standard	*Number and Operations.* The article describes ways to help students understand relationships among numbers and make reasonable estimates.
How to use	The article provides traits of good estimation discussion questions. It also highlights how teachers can use students' estimations to help target any misconceptions they might have about numbers and operations. Estimation also can be used to practice and reinforce number concepts and to develop strategic thinking.
Grade levels	• Elementary • Middle school

Research	Anecdotes or suggestions based on practice or from experts in field

Romberg, T.A. (2001, October). Mathematical Literacy: What Does it Mean for School Mathematics? *Wisconsin School News,* 5-8, 31. http://www.wcer.wisc.edu/ncisla/publications/articles/OctMathWASB.pdf

What it's about	This article describes literacy in the language of mathematics, relating it to literacy in any language. It emphasizes the importance of not only learning concepts and procedures but being able to use these to solve non-routine problems in many different situations. It then discusses implications for mathematics curriculum that teaches mathematical literacy.
How it addresses standard	*Problem-Solving.* The article emphasizes solving problems that arise in different contexts. *Communication.* The article gives examples of ways that students can learn to become mathematically literate, or use the language of mathematics.
How to use	Examples are provided by teachers of classrooms that emphasize mathematical literacy. There also is an example of a general strategy for problem-solving that can be used to help students to approach problems in different contexts.
Grade levels	• Elementary • Middle school • High school
Research	Anecdotes or suggestions based on practice or from experts in field

Rowan, T.E. (n.d.). *The Geometry Standards in K-8 Mathematics.* Retrieved July 13, 2004, from http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-002184-2184,00.shtm

What it's about	Many students have an incomplete understanding of geometric shapes and their characteristics. This article suggests that this is due to limited opportunities to explore and discuss these characteristics. The article provides suggested activities for grade levels kindergarten through 4 and 5 through 8.
How it addresses standard	*Geometry.* It describes activities to support students in analyzing characteristics and properties of two-dimensional shapes.
How to use	The article includes several activities for grades k-4 and 5-8. The information on how students come to understand the invariant characteristics of specific shapes is also useful for monitoring student progress.
Grade levels	• Elementary • Middle school
Research	Anecdotes or suggestions based on practice or from experts in field

Silver, E.A., & Smith, M.S. (1997). *Implementing Reform in the Mathematics Classroom: Creating Mathematical Discourse Communities, Reform in Math and Science Education:*

Issues for Teachers. Columbus, OH: Eisenhower National Clearinghouse. http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-004816-4816_01

What it's about	This is a chapter from a book on educational reform in mathematics and science. It describes the important role that communication and discourse have taken on in mathematics education, and describes ways to support these skills in the classroom. It also emphasizes the importance of allowing students to develop their own strategies to solve problems and to be able to justify these strategies mathematically. Students are engaged in mathematical thinking, rather than memorizing procedures, and their explanations are used to guide and modify instruction.
How it addresses standard	*Problem-Solving.* It focuses on helping students apply and adapt a variety of appropriate strategies to solve problems. *Communication.* It emphasizes the importance of students' abilities to communicate their mathematical thinking and evaluate the strategies of others.
How to use	The article gives examples of ways to increase class discussion in mathematics, including sources for problems to facilitate discussion. It also describes and gives examples of the potential problems teachers may encounter when implementing this approach.
Grade levels	• Elementary • Middle school • High school The chapter focuses on middle school, but the information is applicable to all grades.
Research	Anecdotes or suggestions based on practice or from experts in field

Tiernery, C., & Storeygard, J. (2004). *Including All Students in Class Discussions.* Retrieved July 13, 2004, from http://www.lab.brown.edu/investigations/author/q30.html

What it's about	This is the answer to the following question posed to two researchers in the Accessible Mathematics Project: "I want to include all students in class discussions, but some of my special education students tune out during meetings. What can I do to make them feel included?" It provides descriptions with examples of researchers' observations of effective ways that students with special needs are included in the general education math curriculum at the elementary level. The main focus is on facilitating class discussion.
How it addresses standard	*Communication.* It gives examples of questions that facilitate students' communication of their mathematical thinking.
How to use	Although the research is focused on a specific mathematics curriculum (Investigations), the suggestions and examples based on their research could be applied to any curriculum. They emphasize that, although they are focusing on students with special needs, the teachers in the study did not limit their discussion of inclusion to the special education students.

Grade levels	• Elementary
Research	Anecdotes from research study

Vacc, N.N. (1993). Questioning in the Mathematics Classroom. *Arithmetic Teacher, 41*(2), 88-91. http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ENC-002236-2236,00.shtm

What it's about	This article describes research on the types of questions teachers typically use. It then identifies three categories of teacher questions (factual, reasoning, and open), and provides examples and uses of each type.
How it addresses standard	*Reasoning and Proof.* The article describes the types of questions that support students in making conjectures and developing and evaluating mathematical arguments. *Communication.* Through questioning, teachers support students in communicating their mathematical thinking.
How to use	The article provides several examples of the different types of questioning that support student understanding, using geometric shapes for the examples. It points out that research shows that factual questions predominate teaching. Teachers might want to reflect on the types of questions they ask as a part of their instruction.
Grade levels	• Elementary • Middle school • High school
Research	Suggestions based on research

Weideman, W. (1995). Problem Solving Math Class: "Word Problems" Were Never Like This. *Middle School Journal, 27*(1), 11-17. http://www.enc.org/professional/learn/research/journal/math/document.shtm?input=ACQ-107228-7228,00.shtm

What it's about	This article discusses difficulties that students experience in problem-solving, using the reflections of pre-service teachers as examples. The reflections also highlight concerns regarding how problem-solving is taught. The article ends with suggestions on ways to teach problem-solving in the middle schools.
How it addresses standard	*Problem-Solving.* The article focuses on applying and adapting a variety of appropriate strategies to solve problems.
How to use	Teachers can use the descriptions of poor instruction in problem-solving to analyze their own teaching. The article includes specifics on how to teach problem-solving, including several examples. It also includes a description of the characteristics of a good problem.
Grade levels	• Middle school
Research	Suggestions based on research

Witzel, B., Smith, S.W., & Brownell, M.T. (2001). How Can I Help Students with Learning Disabilities in Algebra? *Intervention in School and Clinic, 37*(2), 101-104. http://www.ldonline.org/ld_indepth/math_skills/algebra_and_ld.html

What it's about	This article focuses on the difficulty many students have in connecting what they learn in algebra with previously learned arithmetic concepts. Although the article was written for teachers of students with learning disabilities, the description of how arithmetic and algebra are connected is appropriate for all.
How it addresses standard	*Algebra.* It describes how to help students learn to represent and analyze mathematical situations using algebraic symbols.
How to use	Using an explicit instruction approach, the article gives recommendations to guide teachers in "bridging the gap" between arithmetic and algebraic concepts. In addition, it presents a chart that can be helpful in determining where a student may be having difficulty. It gives a brief description of teaching through the sequence of concrete to representational to abstract (CRA) as a way of making instruction more explicit.
Grade levels	• Middle school
Research	Anecdotes or suggestions based on practice or from experts in field