Project-INSPIRE Grades 2-5 Course 2 Lesson 2 (33:00)

SPEAKER 1: Welcome to Grades 2 to 5: Nemeth Code Symbols for Fractions and Spatial Problems, Instructional Tools, Materials, and Technology. This is "Lesson 2: Spatial Arrangements." Let's look at slide two, which has the objectives. You're going to be able to read and write addition, subtraction, and multiplication problems that are spatially aligned, and these are going to include problems that have multi-digits, with and without commas, decimals, money, and, our friends simple and mixed fractions.

So let's get started on slide three by talking a little bit about spatial arrangements, or what we tend to think of as vertical problems. And first, I'd like to focus on addition and subtraction. So on the left, I have my very basic addition problem: 280 plus 15 equals 295. When I go to put this problem in braille, just as I do in print, I need to align my ones, my tens, my hundreds. Nothing changes there.

In spatial problems, there is no ever, ever use of the numeric indicator. If I'm doing addition and subtraction, I need to look to see how wide my problem is above the line, above the separation line, and I'm going to put my addition symbol, dots 3-4-6, or my subtraction symbol, dots 3-6, one cell to the left of the widest number above the separation line.

Now, that separation line that separates the problem from the answer is made with dots 2-5. How do I know the length of that separation line? Well, I have to plan for my problem, including my answer, if I am brailling that. Not for the student to braille it, but if I am brailling it. And, because, that's because you'll see in just a moment that my separation line has to go one cell to the left, and one cell to the right past the widest part of the problem. And this means I'm going to have to do some planning.

With spatial problems, you have to leave a line above and below the spatial problem. And we'll get more into this during the lesson. So let's go on to slide four and look at the anatomy of a spatial problem. So I have the problem 814 minus 25 is 789. So let's start with the minuend in this case, which is 814, that top number. So I'm going to braille that. Then I'm going to think about where does my minus sign go in this case?

So I look at the top number, 814, and I see that it is three cells. My 25 is only two cells. So I know that my minus sign has to go one cell to the left of the widest part of the problem. So it's going to go one cell to the left of the 814. Now I have to line up my columns. And notice how that 25 is not right next to the minus sign. Rather, the 20 is under the 10, and the 5 is under the 4. And then you'll see I do my separation line.

Notice, again, that I mentioned before, my separation line goes one cell to the left, and one cell to the right of the widest part of the problem. So since the widest part of my problem takes four cells, that minus, space, 25 is four cells, I know my separation line is going to be six cells in length. Notice how my answer lines up perfectly underneath the ones under the ones, the tens

under the tens, and the hundreds under the hundreds. So my 789, which is my answer, keeps the place value of the problem.

Let's go ahead and go on to slide five. Let's take a look when I add commas into the mix. So I've got 56,780 plus 12,237. This time I don't actually have an answer. I didn't get an answer. You're going to braille problems with answers, without answers. So let's practice both ways today.

So when I go to braille this problem, I start out with my addend, my top number, 56,000, got my comma dot 6, 780. Then I'm going to look to see what's the widest part of the problem. Well, both addends are the exact same length. So no problem. We're gonna put that plus sign. And then I'm going to braille 12,237 with my comma. So notice how those commas are aligned. That separation line, again, goes one cell to the left, and one cell to the right of the widest part of the problem.

Let's take a look at some more examples here on slide six. When you have our spatial arrangements, or a vertical problem side by side, you need to either insert one or two cells between separation lines. Now, many of our students prefer two cells, so you want to experiment with your student to see what they prefer.

So I have three problems here, all in spatial vertical format. 651 plus 80; 9,027 minus 39, notice those commas; and 5,628 minus 2,407. I'm going to ask you to go ahead and look at the first one on your own. Notice how I lined everything up. Notice that my separation line is six cells in length, because the widest part of my problem is four cells.

Let's look at the middle problem together. That's 9,027 minus 39, with that comma in there. So I'm going to begin by brailling 9,027, including that comma, which is dot 6. Now, I know I have to put my subtraction sign to the left of the widest part of the problem. Well, obviously, 9,027 is much wider than 39. So I'm going to put that minus sign one cell to the left of the 9 in the 9,027.

So I'm down on the second line. I put that minus sign. I am cognizant that I need to line up my columns. So in this case, I'm going to space over, hitting my space bar three times. And that will put me ready to braille my 3, which represents 30, right underneath that 2, which represents 20 in 9,027. And then I'll braille my 9.

I've counted up how many cells I need for my separation line. And so I'm going to go ahead, and I'm going to braille the correct number of cells. So why don't you figure out how many cells do I need, and then check me. Did I actually get eight? I hope I did. Alright, I'd like you to pause for a second, and I'd like you to go over the third problem I have here: 5,628 minus 2,407. So go ahead and take a second to look this problem.

Okay, so did you pick up that: A, I aligned my commas, B, my minus sign went to the left, one cell to the left, of that bottom number of 2,407, and did you check out to make sure that I did my separation line properly? Now, if you're not comfortable with spatial problems as far as brailling them for students, this would be a great time to practice brailling these three problems, and then check your work against mine.

Let's go ahead and go on to slide seven, because now it's time for you to do Activity 2A, which is to interline the spatial problems. So I have four problems for you to interline. So go ahead and pause. And when you're ready, please come back.

Okay, slide eight has the answer key for Activity 2A. Did you write out the problems properly? Did you get your commas aligned? Your minuses and plus signs? Everything lining up? Okay, I hope so. Again, if you feel like you need some more practice, you can go ahead and braille these. I'm going to go ahead, though, and go on to slide nine, which is actually giving you a chance to do some brailling. I want you to transcribe on the three addition and subtraction problems that I have here in print. So when you're ready, go ahead, and come on back.

Alright, slide 10. How did you do brailling these problems? Are your separation lines the same length as mine? Did you leave one or two cells between the first and the second problem, and the second and the third problem? Did you make sure that your plus sign or your minus sign is one cell to the left of the widest part of the problem?

Let's go ahead on to slide 11 and talk about when our problems are numbered. In this case, I have directions that say, "Add or subtract," and then I have four problems, problems 35, 36, 37, and 38. So I braille my directions in UEB, starting in cell five. I chose to put my opening Nemeth indicator right after my directions.

I could have skipped a blank line and put that open Nemeth indicator on a line by itself, also followed by a blank line, because with spatial problems, I have to leave a blank line above and below the problems. Now, notice with problem 35, I begin with my problem number: numeric indicator 35, punctuation indicator, period. I then am going to plan for a space, so that there is a space between the period and my separation line. Even though they're not on the same line, I have to do that planning. So where am I going to put my 32? I have to account for the plus 20, and I have to account for that the separation line goes one cell to the left, and one cell to the right of the widest part of the problem.

I'm leaving two cells between the end of the separation line, and when I begin the problem number 36. So I do 36, punctuation indicator, period. I'm giving enough space for my separation lines so that my separation line doesn't come right under that period. That's a mistake we often see. So you need to give space for that. I'm going to braille my 94 minus 7. Remember, the minus sign goes one cell to the left of the widest part of the problem.

I leave a blank line between problems 35 and 36, and before I start problem 37. So problem 37, start with the problem number. Account for where that separation line is going to go, so I can plan where to braille 403. Next line, minus, 246, and then my separation line.

We need to leave a space or two spaces between the end of the separation line and the numeric indicator for the problem number 38. 38, punctuation indicator, period. And then I need to make sure I leave enough space for my separation line to not come right under that period and the problem number, but instead that we have one blank cell.

Now, in this particular case, though we don't see it, what follows these math problems are some more text, let's say some more directions. So I have to terminate Nemeth code, because I am done, and I'm going back to UEB. So my Nemeth code terminator goes on a line by itself. Now, remember, I have to leave one blank line after spatial problems. So I'm going to leave a blank line. And then in cells one and two, I'm going to do 4-5-6, 1-5-6.

Let's go on to slide 12, and talk about spatial arrangements for multiplication problems. There's a lot of commonalities between a multiplication problem and either an addition or a subtraction problem in spatial format. Of course, you're going to align your numbers by columns.

You're going to use dots 2-5 for your separation line, and put it one cell to the left, and one cell to the right of the widest part of the problem. And you're going to remember that blank line above and below the problem. But the big difference here is that the multiplication cross goes one cell to the left of the multiplier. I like to think of it as hugging.

On slide 13, let's take a look at the anatomy of a multiplication problem. I've got 8,207 times 6. So I'm going to braille 8, comma, 2, 0, 7 in this case. So that's my minuend, or my top number. Now, when I go to braille my multiplier, I'm multiplying by 6. So I know that 6 needs to line up under the 7 in 8,207. And I know that that multiplier and that multiplication cross have to hug.

So I know it takes two cells to braille my multiplication cross, so therefore, I'm going to start it under the 2, as in the 200, 8,207. So dot 4, 1-6, and then my 6. So when I look at how to design the separation line for this problem, I have an answer here of 49,242. That's my product. I need to be sure that my separation line is long enough so I have one cell on the left, and one cell on the right that is longer than that separation line.

Let's go ahead and look at slide 14, where I give you some examples of multiplication problems. I actually have four of them here. So 5 times 4, real simple problem. Notice that it takes three cells to write the multiplier and that multiplication cross. So my separation line is going to be five cells.

In 982 times 3 is 2,946, I need to look at the widest part of the problem. It happens to be my product, 2,946. So again, one cell to the left, and one cell to the right of the widest part of the problem.

Now, that third problem, 500 times 8 is 4,000. It looks kind of funny, doesn't it, because we are having the answer of 4,000 with a comma in it. That takes five cells. So my separation line has to be seven cells. So it looks a little lopsided to me.

But again, my goal is to line up my ones, my tens, my hundreds, as I did, and to ensure that my multiplication cross is hugging up against my multiplier. So the top part of the problem, it's a little bit over to the right. But that's because I'm accounting for needing a separation line that's long enough to go one cell to the right, and one cell to the left of my product.

So let's look at the fourth example, 63 times 74 is 4,662. I have my 63. Going to put my dot 4, 1-6 for my multiplication cross. And then I need to line that 74 right up under that 63. So now I

have, four cells is the widest part of the problem above the separation line. It also happens to be the widest part of the problem below the separation line where I put my product. So therefore, I'm going to have a six-cell separation line.

So you can see there's a lot of planning that needs to go on here if you're brailling a row of problems for your student. Now, I really want to stress to you. Please do not penalize your student if their separation lines are a little bit off, especially if they're having to do that product. They don't quite know what the product's going to be. That's why they're learning to multiply, folks. So we don't want to penalize them for setup.

You do want to work with them to get those ones and tens, you know, and hundreds lined up, but again, recognize that if they're doing the multiplication correctly, we might cut them a little slack. Many of our students will be doing the multiplication on their abacus, and then brailling their products. We still want them to have definite practice reading spatial arrangements of problems. And they should still have the opportunity to braille some of these out.

Let's take a look at slide 15, where we're going to start to talk about problems with money. So we have our decimal points and our dollar sign. So our dollar sign is braille dots 4, 2-3-4, takes two cells, and our decimal point is dots 4-6. Sometimes in print, the dollar sign has the plus or minus sign right underneath it. Sometimes a plus or minus sign is to the left.

Our job is not to make judgments on how people prepare things in print, but instead, to focus on giving our braille reader the same information. So you're going to follow the print on where you place your dollar sign, and your plus and minus sign in relation to that. The important thing to remember, too, is that our decimal places need to align, just as our commas did before. So let's go on to slide 16, and I'll show you some examples.

I have three problems that use decimal points, two of which are money problems. And my job is to follow the ink print. So let's look at problem 5, which is our first example. $4.82 minus $2.10. Now, notice in the print, the minus sign is to the left of the dollar sign. So I'm going to start out by thinking about how long my separation line is going to be, so I can decide where to braille my top number.

Figured that out, so I'm going to go ahead and braille my dollar sign, 4, decimal point, 82. Now, in print, the minus sign is to the left of the dollar sign. So you notice how I've done the same thing in the braille. And then I'm going to line up my $2.10 right under my $4.82. So do you see how those decimal points align? And that separation line is going to go one cell to the left of my minus sign, and one cell to the right of the numbers.

Problem 6, I don't have a decimal point in this one. My job is to follow the ink print. I'm not questioning why the teacher doesn't have any dot 00s. Not my job. My job is to prepare the braille for the braille reader. So I have my dollar sign, 900. Now, in this example, the plus sign comes right underneath the print dollar sign. So I'm going put my plus sign right underneath the second cell of the dollar sign, or the "s". And notice how my 75 lines up. Now, I accounted for how many cells my separation line needs to be. So I have one cell to the left, and one cell to the right for the widest part of the problem.

Now, I forgot to point out to you, but I'm sure you noticed. After problem 5 and before problem 6, I made sure I left one cell after my separation line, and before the start of my numeric indicator. And I did the same thing here, between problems 6 and 7. I left one cell after that separation line before I started the numeric indicator to number the problem 7.

And in this problem, I have a minus sign. There is no dollar sign. This is just straight decimals. 19.60 minus 10.20, so everything lines up, as far as my columns and my decimal point. And that minus sign goes one cell to the left of the widest part of the problem, in this case, both equal, and then that separation line.

So there's a formula here. The biggest challenge, both for you and your students, is brailling these accurately when it comes to spatial problems. They're not hard to braille. It's just you have to account for that separation line. And if you're numbering problems, where those problem numbers are going to go.

Let's take a look at slide 17, where you have three more examples of spatial problems that use dollar signs and decimals. So notice how I've got everything lined up nice and neat. Notice that in problems 1 and 2, my plus sign and my minus sign are one cell to the left of the widest part of the problem. And in problem 2, I accounted for where do I need to do that for including the answer, the difference between 6.24 minus .63.

Let's go over problem 3 together. I've got $3,015 times 2 is $6,030. Now, I'm going to start off by brailling $3,015. Then I'm going to braille my multiplier with my multiplication cross, and I need to remember that those hug. So when I go to start my multiplication cross, it's going to start under the 0, which is representing zero hundreds. And my multiplier is going to line right up, that 5 is going to be right above the 2, which is my multiplier.

Both my multiplicand, or my top number, and my product are the same length. So I'm going to take that into account when I braille my separation line. And, of course, you can see that my dollar signs, my numbers, and my commas all line up.

Let's go ahead and go on to slide 18. We like to give you examples. So slide 18 gives you two more examples with dollars and decimals. So my first one is $61 times 40. Just as we talked about, I'm going to have my dollar sign, 61.00. On the next line, that multiplier is going to hug right up against that multiplication cross. So I'm going to actually be getting that multiplication cross under the 1 in $61. And so dot 4, dot 1-6, and then 4, 0.

Notice that separation line, again. I'm going to keep pointing that out, though, that it goes one cell to the left, and one cell to the right of the widest part of the problem. I'll let you look at 5.2 times 6 on your own, and make sure that I did it right, and that yes, indeed, my separation line was to be five cells.

Let's go on to slide 19, and it's time to put you to work with Activity 2C. Interline the four problems, please. When you're ready, come on back. Alright, let's take a look at slide 20, where we have the answer key. So I want you to check that you wrote out the problems the exact same

way I did, and that you numbered them properly, that all your columns line up, and that you're good to go.

Alright, slide 20. Whine, now. It's time to do a little more work. You've got to transcribe three problems for me. Two have dollar signs, and one has decimals, as well. So you've got a lot of work to do here. Go ahead and transcribe these on slide 21. When you're ready, come on back.

Alright, slide 22. Did you transcribe your problems properly? Are your dollar signs where my dollar signs are? Are your decimal points where my decimal points are? How about our separation lines? Did we get the same number of cells? Did you make sure that you left one cell after the end of the separation line, and before you started the next problem number? If you mixed up a little bit on some of these things, why don't you try brailling these problems looking at my model here on slide 22.

Slide 23 is our last topic for Lesson 2. And that's talking about spatially aligned problems that have simple fractions. In just a minute, we'll talk about mixed numbers. When you go to set up a spatially aligned problem, you're going to set it, that has fractions, you're going to set it up the same way. But there's just a little bit of a difference.

First, you have to have your open fraction indicators, your fraction lines, and your closing fraction indicators. And folks, these must, must, must be aligned. And this gets a little tricky, as I'm going to show you on the next slide. But for right now, let's just look at example 1, one-half minus three-eighths.

So I'm going to braille my problem number 1. Going to make sure I leave enough space to account for my separation line. I'm going to open up my simple fraction, 1-4-5-6, numerator 1, my horizontal fraction line, dots 3-4, my denominator 2, going to close that simple fraction, dots 3-4-5-6.

Now, my minus sign goes one cell to the left of the widest part of the problem. So it's going to go to the left of the open simple fraction indicator. Then I'm going to do my 3, my horizontal fraction line, my 8, and I'm going to close the fraction. Notice how I've left enough room for my separation line.

Now I'd like you to go ahead and look at one-seventh plus four-sevenths, and make sure I lined up everything just the way I was supposed to. And when you're ready, we're going to go ahead and go on to slide 24. So take a second, and look at these two problems, if you need to. You may even want to braille them if you're not used to brailling fractions in spatial format.

Now, slide 24 introduces us to mixed numbers and how we braille these in spatial format. It's not hard. The whole number, the fraction indicators, so those opening and closing, the fraction lines, all have to be aligned. Let's go ahead and look at the problem ten and four-fifteenths plus nine and seven-fifthteenths.

So what I have to do here is I have the horizontal fraction lines, so that's going to take one cell, dots 3-4, I'm going to start out with my whole number 10, open up the mixed fractions, so that's

4-5-6, 1-4-5-6, my numerator is 4, dots 3-4 for that horizontal fraction line, 15 for my denominator, and then I'm going to close that mixed number with 4-5-6, 3-4-5-6.

That plus sign is going to go one cell to the left of the widest part of the problem. So notice I do plus, and then I have a space, and then I have the 9, because that 9 needs to line right up underneath the 0 in the 10. I can't lose place value here, just because I'm doing mixed numbers or simple fractions.

And then I just follow along with my pattern so that everything lines up beautifully. I'm going to open the mixed number, fractional part, my 7, which is my numerator, my fraction line, my 15 for my denominator, and I'm going to close out. And of course, by now, we all know that the next thing I'm going to say is that that separation line is one cell to the left, and one cell to the right of the widest part of the problem.

Slide 25. This is where it gets a little tricky. And people really struggle with what happens when my numerator and denominator don't exactly line up. Well, it's not hard, because the rule is my opening and closing fraction indicators and my fraction line have to line up. Think of it this way: fraction lines, whether they're horizontal or they're diagonal, they need a hug. So you always have to make sure your numerator and your denominator are touching up against your fraction line.

Let's see what I mean here. I have one and three-fifths plus five-tenths. Now, just looking at the print, I notice that my denominator, 5, is only going to take one cell, but my denominator, 10, is going to take two cells. Better account for this when I am setting up one and three-fifths. So I do 1, then I open up the mixed fraction part, 4-5-6, 1-4-5-6, I do my 3, I do my horizontal fraction line, and I do my 5.

But I know that 10 is going to be my next denominator. I have to account for that 0 that's going to be in the 10, that's going to need to go right below that blank space. So I'm going to, after I braille my 5 in my denominator, in one and three-fifths, I'm going to do a space, and then I'm going to close my fraction part of the mixed number.

So when I go to do the second mixed number, which is five and one-tenth, that plus sign goes to the left of the 5. I'm going to open up the mixed fraction part. I'm going to do my 1, I'm going to do my fraction line, and notice the 10 fits right in there, just like it should, and then I close my fraction with 4-5-6, 3-4-5-6.

So I have to do some accounting to make sure that everything's going to line up just perfectly. So my next example, five and eleven-fifteenths minus four and two-fifteenths. So I braille my five and eleven-fifteenths. No problem. I'm good at that. So let's look at the second part of this problem: minus four and two-fifteenths. I'm going to braille my minus sign, and then my 4, my whole number 4.

I'm going to open up the fractional part of the mixed number, 4-5-6, 1-4-5-6. The numerator this time is 2. I'm going to put a space, and then the 2, because that 2 needs to hug right up against that horizontal fraction line, and then I'll put my denominator 15, and then I will go ahead and

close the fractional part of the mixed number. And of course, my separation line is one cell to the left, and one cell to the right.

Now, my last example on this slide. Wow! I have two and one-eighth, but then I just have a simple fraction that I'm adding. It's not another mixed number. So I have to account for that missing mixed number by leaving a space before I open up the fraction. Oh, but wait. I've got a simple fraction. So that's not hard. Look what I did. At first, I brailled out two and one-eighth. You guys know how to do that now. Then I said, wow, I've got a simple fraction of three-fourths.

Well, the first thing I need to deal with is my plus sign. My plus sign goes one cell to the left of the widest part of the problem. So it's going to go one cell to the left of where the 2 is in that first mixed number. There is no whole number. So I'm going to put a space there. I'm dealing with a simple fraction, folks, not a fractional part of a mixed number. So I don't need the 4-5-6.

There's another space. Now I'm going to do 1-4-5-6 to open up my simple fraction. And notice how it lines right up with the 1-4-5-6 in the mixed number. I'll do my numerator 3, I'll do my diagonal fraction line, and then I will do my 4, which is my denominator. It's really tempting that most people want to close a simple fraction here, but remember, we must, must, must line things up. So I put a space, and then dots 3-4-5-6. And, of course, that separation line is why. That's a tricky one. You might want to braille it.

Alright, slide 26. It's your turn. You get to braille these in Activity 2E. So please braille problems 3, 4, and 5 here in Activity 2E. And when you're ready, come on back.

Alright, slide 27 has the answer key for Activity 2E. Did you braille your mixed numbers and simple fractions properly? Did you get your horizontal and diagonal fraction lines lined up? How about your whole numbers, when you had mixed numbers? Check the length of your separation line. If you put more than one problem on a line, did you make sure that you separated your problems properly?

Lots to check out here. So once you know that you've done Activity 2E properly, congratulations! You are done with Lesson 2. And you're ready to now move on to Lesson 3, where we're going to talk about division, when it is set up in a spatial format. We don't want to forget our other operation here. So you've just done addition, subtraction, and multiplication in spatial formats. Time to move on to division. Thank you very much.