Chapter 5: Fractions and Mixed Numbers

New Symbols Introduced

⠰ [dots 1-4-5-6] Opening simple fraction indicator (no print equivalent)

⠸ [dots 3-4-5-6] Closing simple fraction indicator (no print equivalent)

⠤ [dots 3-4] Horizontal fraction line (–)

⠰⠤ [dots 4-5-6, dots 3-4] Diagonal fraction line (/)

⠰⠰ [dots 4-5-6, dots 1-4-5-6] Opening mixed number fraction indicator (no print equivalent)

⠰⠸ [dots 4-5-6, dots 3-4-5-6] Closing mixed number fraction indicator (no print equivalent)

Key Points

- **Simple fractions** begin with the opening simple fraction indicator. The closing simple fraction indicator ends a fraction. There are no print equivalents for these symbols.
- In a simple fraction, follow the print and use either the **horizontal** or **diagonal fraction line** to separate the numerator and denominator. The diagonal fraction line is sometimes called a slash.
- When a simple fraction includes a **diagonal fraction line**, pay attention to position of the numerator and denominator to know how to braille the simple fraction.
- **Mixed numbers** combine a whole number and simple fraction.
- After writing the whole number, use the **opening mixed number fraction indicator** to begin the fractional part of the mixed number.
- End the fractional part of the mixed number with the **closing mixed number fraction indicator**.
- **Variables** are unknown numbers written as letters in both print and Nemeth Code.
- Simple fractions and mixed numbers are in Nemeth Code and must be placed within opening and closing Nemeth Code indicators.
- Try to avoid dividing a fraction or a mixed number between lines in word problems. If you must divide an equation, do so at the sign of comparison.

- If there is an abbreviated measurement unit with the simple fraction or mixed number (e.g., in, km), it must be included within the Nemeth Code opening and closing indicators with no contractions.
- Example problems must be included when transcribing materials. Leave a blank line above and below the example problem. In addition, format the example problem like the surrounding problems.

Introduction

Students are usually introduced to simple fractions in third grade and mixed numbers in fourth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015). During third grade, students learn to read and write simple fractions, and by the time they complete fourth grade, they are expected to compare two fractions with different denominators; add and subtract mixed numbers with like denominators; multiply a fraction by a whole number; and use decimal notation for fractions with denominators 10 and 100. In fifth grade, students learn to multiply and divide fractions (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

Simple Fractions

$\frac{3}{4}$ is an example of a **simple fraction** with a horizontal fraction line.

The top part of a simple fraction is the numerator, and the bottom part of the simple fraction is the denominator. The **numerator** is the number of parts being counted, and the **denominator** is the total number of parts in the whole (New Oxford American Dictionary, n.d.).

In $\frac{3}{4}$, 3 is the numerator and 4 is the denominator.

In the Nemeth Code, simple fractions are defined as having numerators and denominators without a fraction in the numerator or denominator (Rule XII, §61). However, simple fractions can include mathematical expressions such as 4+5 and variables such as x.

There are no "tops" and "bottoms" for fractions in Nemeth Code. We have "lefts" and "rights" because the numerator is written to the left of the fraction line and the denominator is written to the right of the fraction line.

Fractions with a horizontal fraction line are enclosed by opening and closing simple fraction indicators (Rule XII, §62a). There are no print equivalents for

these indicators. An **opening simple fraction indicator** is written with dots 1-4-5-6 in Nemeth Code.

> **Teaching Tip:** It may be helpful to point out that the opening simple fraction indicator contains the same dots as the "th" groupsign.

A **closing simple fraction indicator** is written with dots 3-4-5-6 in Nemeth Code.

> **Teaching Tip:** It may be helpful to point out that the closing simple fraction indicator contains the same dots as the numeric indicator.

Similar to UEB, the same dot configuration sometimes represents multiple items in the Nemeth Code, depending on the placement and context of the braille dot configuration.

> **Teaching Tip:** Remind students to always "close" what they "open". Thus, if they begin with an opening simple fraction indicator, they will need a closing simple fraction indicator at the end of the fraction.

A **horizontal fraction line** is written with dots 3-4 in the Nemeth Code.

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> **Teaching Tip:** It may be helpful to point out that the horizontal fraction line contains the same dots as the "st" groupsign.

Example 5.1 contains 6 examples of simple fractions with horizontal fraction lines.

Example 5.1

$\frac{1}{3}$ $\frac{5}{8}$ $\frac{11}{12}$

$\frac{6}{7}$ $\frac{39}{100}$ $\frac{30+50}{100}$

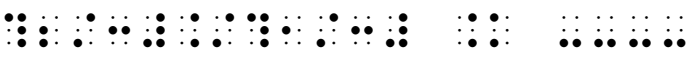
Students learn to generate simple equivalent fractions as well as add and subtract simple fractions with like denominators in third grade and with unlike denominators in fourth grade (Common Core State Standards Initiative, 2010).

Sometimes the problems are in horizontal (linear) format, and other times the problems are spatially aligned (vertical).

Example 5.2 contains eight linear problems that contain one or more simple fractions with a horizontal fraction line.

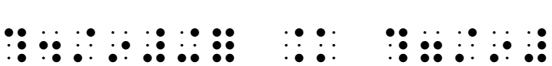
Example 5.2

$\frac{2}{3} \div \frac{1}{3} = \_\_\_\_$



$\frac{5}{6} + 1 = \_\_\_\_$

$\frac{4}{9} + ? = \frac{7}{9}$



$\frac{23}{100} < \frac{71}{100}$

$\_\_\_\_ - \frac{1}{2} = \frac{4}{13}$

$\frac{2}{6} = \frac{1}{3}$

$$\frac{1}{2} \times \frac{4}{5} = \frac{1 \times 4}{?}$$

$$\frac{3}{10} = .3$$

Some simple fractions contain a **diagonal fraction line**, sometimes called a slash.

$1/3$ is an example of a simple fraction with a diagonal fraction line.

It takes two cells to write a **diagonal fraction line** in Nemeth Code. It is written with dots 4-5-6 in the first cell, followed by dots 3-4 in the second cell.

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Example 5.3 contains three examples of simple fractions with a diagonal fraction line where the expression or number in the numerator appears to be raised higher than the expression or number in the denominator.

Example 5.3

$1/3$ $5/4$ $8 + 3/17$

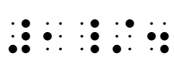
Different rules apply if the numerator and denominator are printed at the same level and the size of print is the same as the surrounding math expressions. In other words, a simple fraction is brailled differently when the numerator is not raised higher than the denominator.

In this situation, simple fraction indicators are not used (Rule XII, §63b). Instead begin with a numeric indicator.

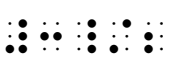
Example 5.4 includes two examples where the numerator and denominator are printed at the same level and the size of print is the same as the surrounding math expressions.

Example 5.4

1/4

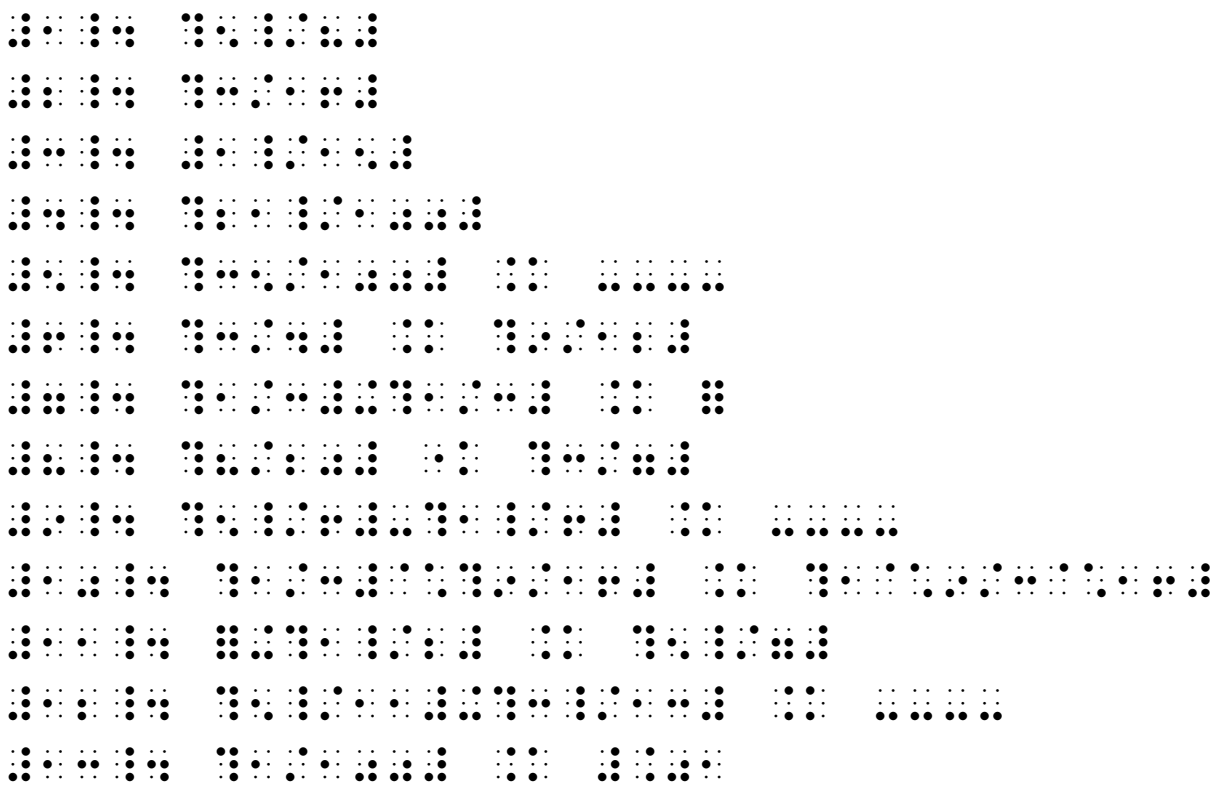


3/2



Practice 5.1

Interline the following simple fractions and problems containing simple fractions. Pay special attention to the fraction line to determine if it is a horizontal fraction line or a diagonal fraction line.



Write the fractions in braille.

1. $1\!\!/_{9}$
2. $\frac{11}{70}$
3. $12\!\!/_{5}$
4. $\frac{1}{3}$

5. $\frac{4}{19}$

6. $\frac{8}{100}$

7. $2\!\!\not\!/\,2$

Write the expressions where the numerator and denominator are at the same level.

8. 1/3

9. 3/7

10. 13/14

Mixed Numbers

A **mixed number** begins with a whole number. It is followed by a simple fraction, comprised of numbers, usually written in smaller print (Rule XII, §64).

$2\,1\!\!\not\!/\,9$ and $5\frac{3}{4}$ are examples of mixed numbers.

In the Nemeth Code, mixed numbers begin with the numeric indicator and the whole number. The fractional part of the mixed number is then enclosed within the **mixed number fraction indicators** (Rule XII, §64). Either the horizontal fraction line or the diagonal fraction line is used within the fractional part (Rule XII, §64).

It takes two cells to write an **opening mixed number fraction indicator** in Nemeth Code. It is written with dots 4-5-6 in the first cell, followed by dots 1-4-5-6 in the second cell. There is no print equivalent for the opening mixed number fraction indicator.

⠸⠸

It also takes two cells to write a **closing mixed number fraction indicator** in Nemeth Code. It is written with dots 4-5-6 in the first cell, followed by

dots 3-4-5-6 in the second cell. There is no print equivalent for the closing mixed number fraction indicator.

> **Teaching Tip:** Remind students to always close what they open. Thus, if they begin with an opening mixed number fraction indicator, they will need a closing mixed number fraction indicator at the end of the fractional part.
>
> **Teaching Tip:** It may be helpful to point out that the same dot configuration is used for fraction lines in the fractional part of mixed numbers as in simple fractions.

Example 5.5 contains five examples of mixed numbers. Pay special attention to the mixed number fraction indicators and fraction line.

If the fraction line is horizontal in print, then a horizontal fraction line is used in braille (Rule XII, §64). Similarly, if the fraction line is diagonal in print, then a diagonal fraction line is used in braille (Rule XII, §64).

Example 5.5

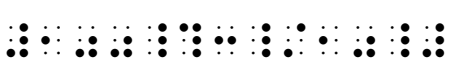
$2\frac{1}{6}$

$3\frac{4}{5}$

$8\ 17/20$

$25\frac{2}{3}$

$100\ 3/10$

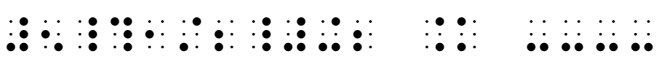


Example 5.6 contains three examples of linear problems that contain one or more mixed numbers. Notice that the previous rules explaining about spacing of linear problems apply when mixed numbers are included in the problem.

Example 5.6

$3\frac{1}{7} < 3\frac{2}{7}$

$5\frac{1}{2} + 2 = \_\_\_\_\_$



$7\ 3/8 = 7\ 9/24$

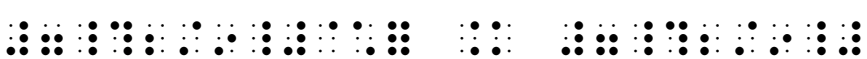
Example 5.7 contains three examples of linear problems that contain one or more mixed numbers and a sign of omission. Notice that any numeral in the mixed number or the entire mixed number may be represented by a sign of omission. Also notice in the first example that a numeric indicator is not needed before the whole number in the subtrahend.

Example 5.7

$9\frac{1}{5} - 2\frac{1}{4} = ?$

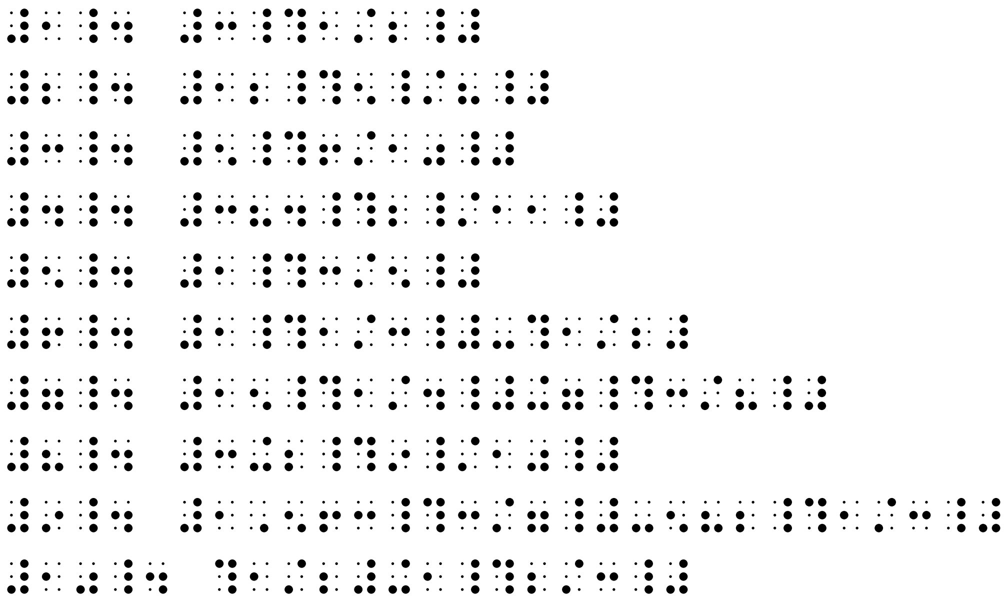
$1\frac{?}{4} + \frac{1}{3} = 1\frac{7}{12}$

$7\frac{2}{9} \times ? = 7\frac{2}{9}$



Practice 5.2

Interline the mixed numbers and problems that contain one or more mixed numbers.



Now write the following mixed numbers and problems containing one or more mixed numbers in braille.

1. $2\ ^1\!/_7$
2. $4\frac{1}{3}$
3. $8\ ^1\!/_2$
4. $1\frac{3}{16}$
5. $3\frac{4}{7}+1\frac{1}{7}=$ _____
6. $?-\frac{1}{4}=2\frac{1}{2}$
7. $4\ ^2\!/_3-3\ ^1\!/_3=?$
8. $12\frac{5}{9}-\frac{2}{?}=12\frac{3}{9}=12\frac{1}{3}$

Spatially Aligned Problems with Simple Fractions and Mixed Numbers

When a spatially aligned problem contains fractions, the fraction indicators and fraction lines must be vertically aligned (Rule XXIV, §178e).

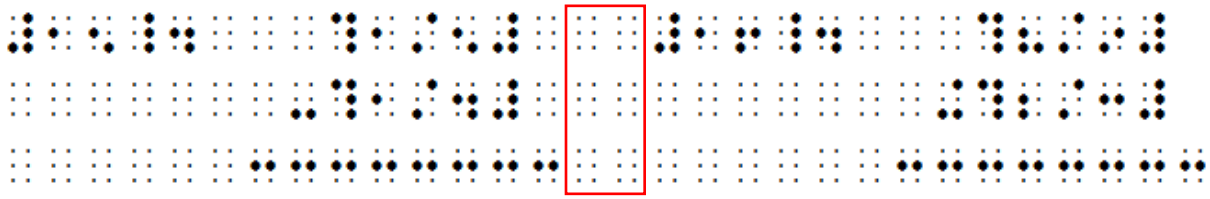
Example 5.8 contains two problems with simple fractions. The simple fraction indicators and horizontal fraction lines are aligned vertically in each problem. Notice that both problems are numbered, and there is one blank column between the period of the problem number and the beginning of the separation line.

All other rules for spatially aligned problems apply. Thus, the minus/plus sign is placed one cell to the left of the opening simple fraction indicator, and the separation line extends one cell to the left and one cell to the right of the widest line in each arrangement.

Example 5.8

15. $\frac{1}{5}$
$-\frac{1}{4}$

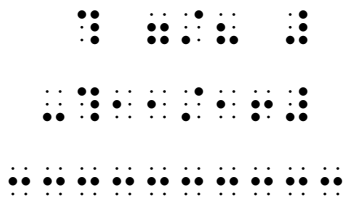
16. $\frac{8}{9}$
$+\frac{2}{3}$



Example 5.9 contains spatial problem with simple fractions. The first fraction has a two-digit numerator and denominator, and the second fraction contains a one-digit numerator and denominator. So that everything is lined up in the problem, the numerators are right justified, and the denominators are left justified (Rule XXIV, §178e).

Example 5.9

$$\begin{array}{r} \frac{7}{8} \\ -\frac{11}{16} \\ \hline \end{array}$$



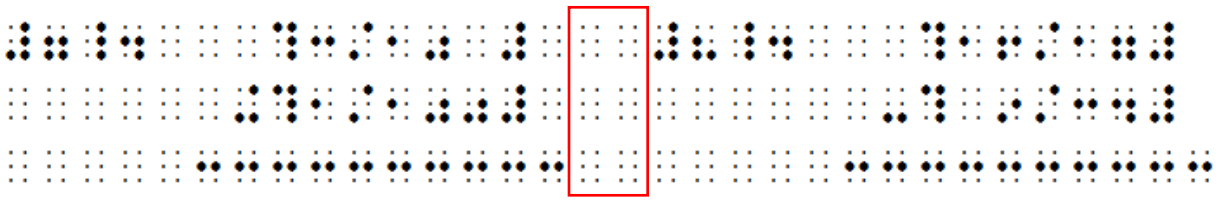
Teaching Tip: Instead of referring to right justified and left justified, it may be helpful to point out that the numerator and denominator will snuggle the fraction line.

Example 5.10 contains two additional numbered problems with simple fractions. Pay special attention to the alignment of the fraction lines and simple fraction indicators as well as the placement of the numerators and denominators.

Example 5.10

7. $\begin{array}{r} \frac{3}{10} \\ +\frac{1}{100} \\ \hline \end{array}$

8. $\begin{array}{r} \frac{16}{17} \\ -\frac{9}{34} \\ \hline \end{array}$



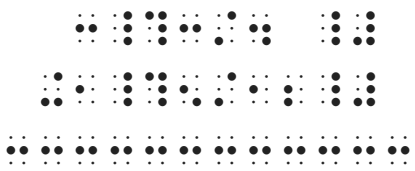
When a spatially aligned problem contains mixed numbers, the whole numbers must also be vertically aligned (Rule XXIV, §178f). All other rules for spatially aligned problems with fractions apply. For example, the numerators of the fractional parts will be right justified and the denominators of the fractional parts will be left justified.

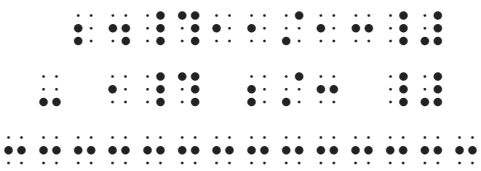
Example 5.11 contains four problems with mixed numbers. Notice how the whole numbers, fraction indicators, and horizontal fraction lines are aligned vertically in each problem. Also notice that the numeric indicator is not to be used in front of the whole number of the mixed number in spatially arranged problems.

Example 5.11

$$\begin{array}{r} 3\frac{3}{4} \\ +\ 1\frac{5}{12} \\ \hline \end{array}$$

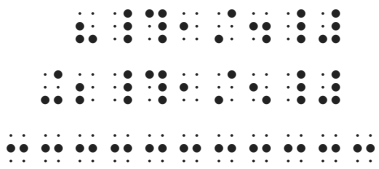
$$\begin{array}{r} 24\frac{11}{13} \\ -\ 1\frac{2}{3} \\ \hline \end{array}$$

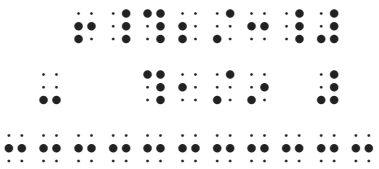




$$\begin{array}{r} 8\frac{1}{4} \\ +\ 2\frac{1}{5} \\ \hline \end{array}$$

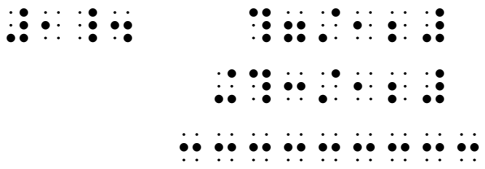
$$\begin{array}{r} 6\frac{2}{3} \\ -\ \frac{1}{9} \\ \hline \end{array}$$

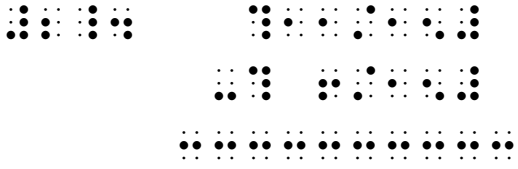


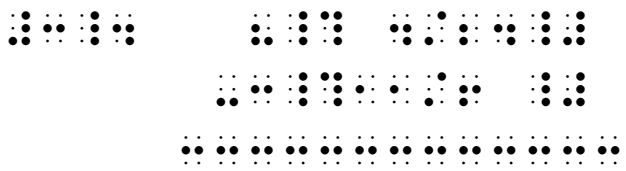


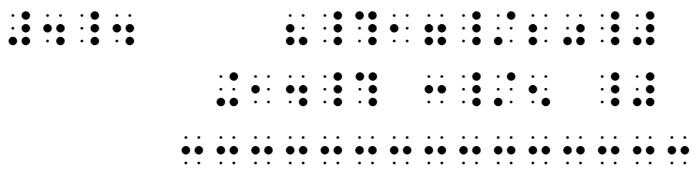
Practice 5.3

Interline the following problems that contain fractions and mixed numbers.









Use your braillewriter to write the problems that contain fractions and mixed numbers in spatial format.

$$\begin{array}{r} \frac{3}{5} \\ -\frac{1}{2} \\ \hline \end{array} \qquad \begin{array}{r} \frac{8}{9} \\ -\frac{2}{9} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{3}{4} \\ +\frac{1}{16} \\ \hline \end{array} \qquad \begin{array}{r} \frac{16}{100} \\ -\frac{1}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 3\frac{5}{7} \\ +1\frac{1}{7} \\ \hline \end{array} \qquad \begin{array}{r} 16\frac{1}{3} \\ -2\frac{1}{4} \\ \hline \end{array}$$

Word Problems with Simple Fractions and Mixed Numbers

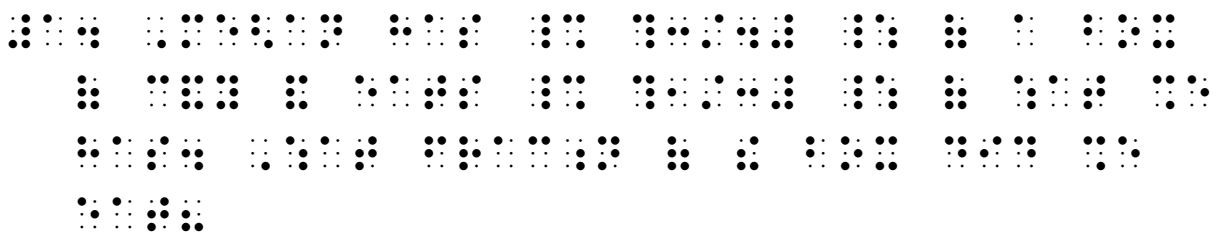
You learned in Chapter 2 that freestanding numbers such as 48 and 100 in word problems are not considered math notation. In contrast, simple fractions and mixed numbers in word problems are considered math notation. Thus, they must be written in Nemeth Code, and Nemeth Code switch indicators must be used (BANA Guidance, 2018, p. 1).

Review the word problems in Example 5.12 and 5.13. Notice that the simple fractions and mixed numbers are in Nemeth Code. They are placed within opening and closing Nemeth Code indicators. Also notice that the opening

Nemeth Code indicator, the math material, and the Nemeth Code terminator are kept on the same braille line when possible.

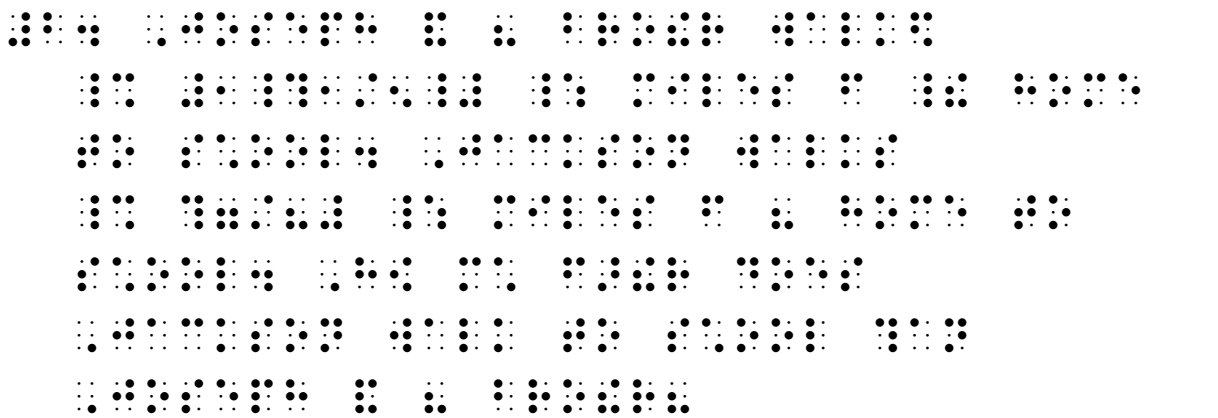
Example 5.12

1. Meghan has $\frac{3}{4}$ of a box of candy and eats $\frac{1}{3}$ of what she has. What fraction of the box did she eat?



Example 5.13

2. Jackson and his brother walked $1\frac{1}{5}$ miles from their home to school. Joseph walks $\frac{7}{8}$ miles from his home to school. How much farther does Joseph walk to school than Jackson and his brother?

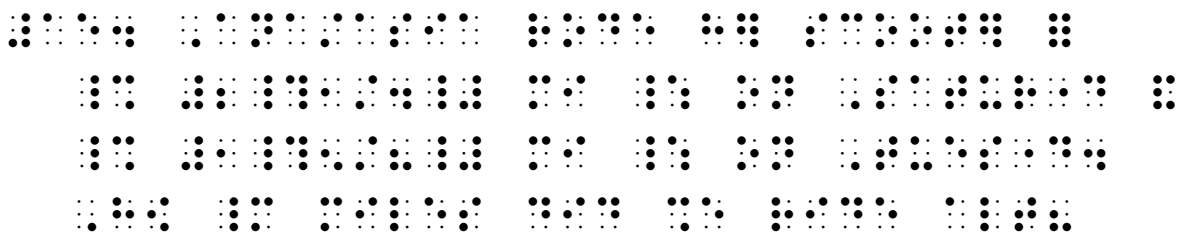


If there is an abbreviated measurement unit with the simple fraction or mixed number (e.g., in, km), it must be included within the Nemeth Code opening and closing indicators with no contractions (BANA Guidance, 2018, p. 8).

Review the word problems in Examples 5.14 and 5.15. Pay special attention to the abbreviated measurement unit and how they are included as part of the material in Nemeth Code. In Example 5.15, also notice how the contraction is not used in the abbreviated measurement unit.

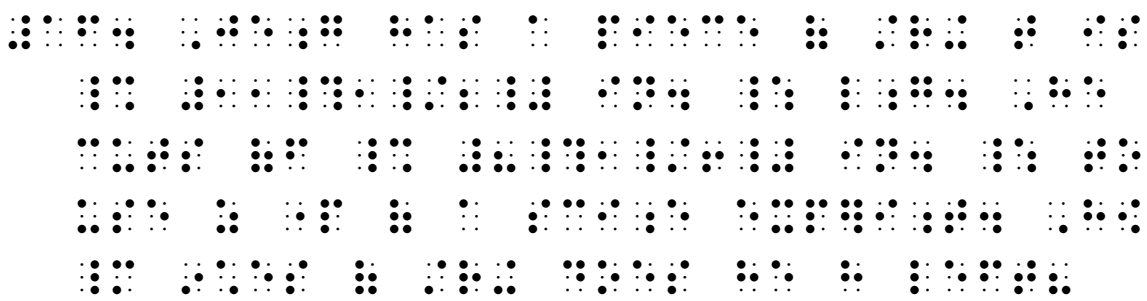
Example 5.14

15. Anastasia rode her scooter for $2\frac{1}{4}$ mi on Saturday and $1\frac{5}{8}$ mi on Tuesday. How many miles did she ride altogether?



Example 5.15

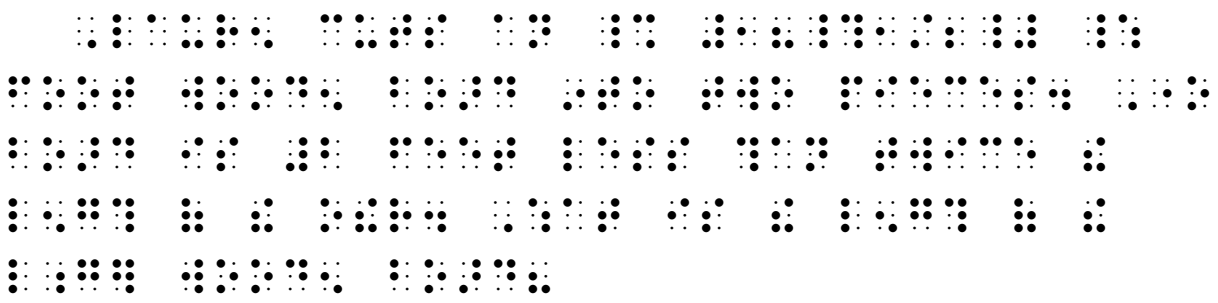
16. Jeong has a piece of string that is $11\frac{1}{2}$ in. long. He cuts off $8\frac{1}{6}$ in. to use as part of a science experiment. How many inches of string does he have left?



In Example 5.16, the measurement unit is not abbreviated. Therefore, it is not included within the Nemeth Code opening and closing indicators. Also notice that the single problem is not numbered. Thus, it treated as a paragraph and begins in cell 3 with run-overs in cell 1.

Example 5.16

Lauren cuts an $18\frac{1}{2}$ foot wooden board into two pieces. One board is 2 feet less than twice the length of the other. What is the length of the longer wooden board?



Practice 5.4

Now write the following word problems in braille.

4. Mr. Stefanik is ordering pizza for the 18 students in his homeroom. If he estimates that each student will eat $1/4$ of a pizza, how many pizzas should he order?

5. Maya was 5 ft. $2\frac{1}{2}$ in. tall last year. She grew $1\frac{1}{4}$ in. in the past year. How tall is she now?

6. Two children weigh $72\frac{1}{3}$ lbs. and $86\frac{2}{5}$ lbs. What is the difference between their weights?

Worksheets with Example Problems

Sometimes worksheets include an example problem. Leave a blank line above and below the example (*Braille Formats*, 10.8.3). Then transcribe the example like the other problems (*Braille Formats*, 10.8.4).

When a colon follows the word "example" do not bold, underline, or italicize it (*Braille Formats*, 10.8.2). However, when no punctuation follows the word "example", follow print formatting (*Braille Formats*, 10.8.1).

If you need to divide a problem, do so at the sign of comparison (Rule XXV, §195e).

The worksheet in Example 5.17 includes an example problem after the directions. Notice how the example begins in cell 1 like the other problems. Also notice that there is a blank line both before and after the example problem.

Since the word "example" is followed by a colon, it is not bolded in braille.

The example math problem will not fit on one braille line. Thus it has been divided across two lines. Pay special attention to how the problem was divided so that the second line begins with a sign of comparison.

Additionally, since the directions are followed by numbered problems, they begin in cell 5 with run-overs in cell 3.

Example 5.17

Fun with Fractions

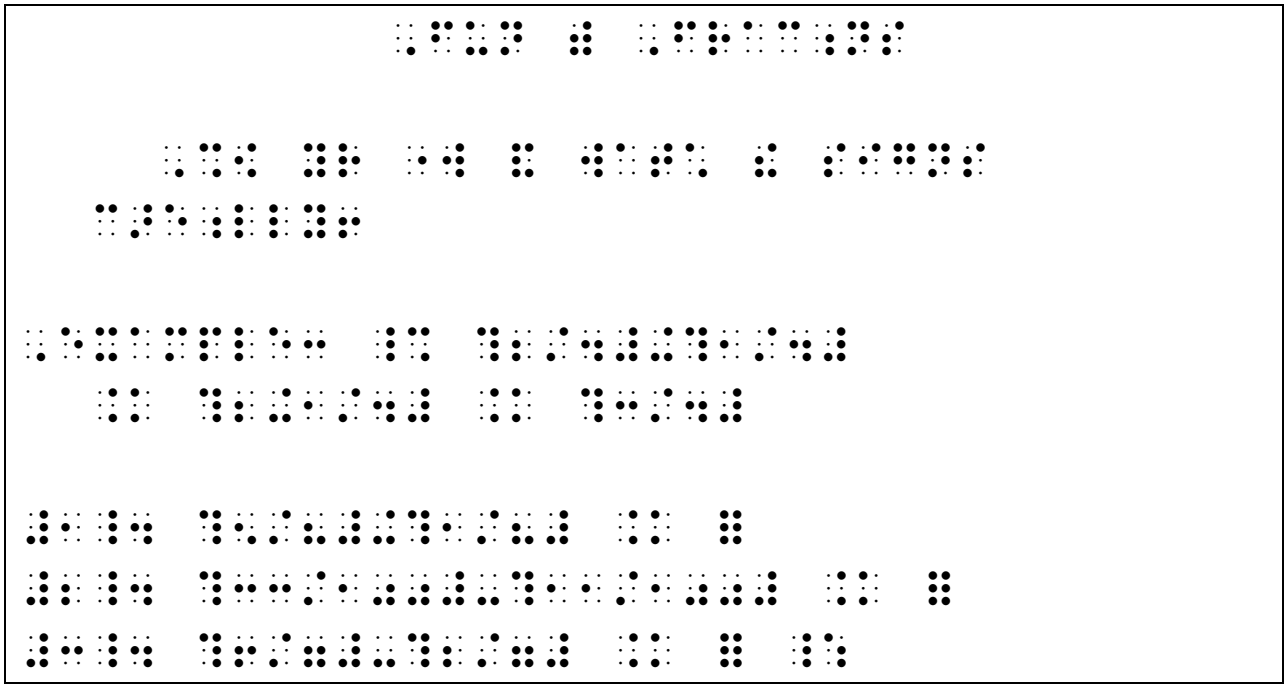
Show your work and watch the signs carefully!

Example: $\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$

1. $\frac{5}{8}+\frac{1}{8}=$

2. $\frac{33}{100}-\frac{11}{100}=$

3. $\frac{6}{7}-\frac{2}{7}=$



Chapter Summary

Nemeth Code Switch Indicators

- The math problem must be on the same line with the opening Nemeth Code indicator and Nemeth Code terminator whenever possible.

Simple Fractions

- Fractions with a horizontal fraction line are enclosed by opening and closing simple fraction indicators (Rule XII, §62a).
- In Nemeth Code, the numerator is written to the left of the fraction line and the denominator is written to the right of the fraction line.
- If the fraction line is horizontal in print, then a horizontal fraction line is used in braille (Rule XII, §64).
- Similarly, if the fraction line is diagonal in print, then a diagonal fraction line is used in braille (Rule XII, §64).

Mixed Numbers

- Mixed numbers begin with the numeric indicator and the whole number.
- The fractional part of the mixed number is enclosed within the mixed number fraction indicators (Rule XII, §64).

Linear Problems That Contain Simple Fractions and Mixed Numbers

- Problems containing simple fractions and/or mixed numbers written in horizontal format in print are written horizontally in braille.
- All other rules for linear problems apply.

Spatially Aligned Problems That Contain Simple Fractions and Mixed Numbers

- The fraction indicators and fraction lines must be vertically aligned (Rule XXIV, §178e).
- The numerators are right justified, and the denominators are left justified (Rule XXIV, §178e).
- All other rules for spatially aligned problems apply.

Variables

- Variables represent an unknown number.
- Write a variable in an equation or expression wherever it occurs.

Word Problems That Contain Simple Fractions and Mixed Numbers

- Simple fractions and mixed numbers in word problems are written in Nemeth Code (BANA Guidance, 2018, p. 1). In addition, Nemeth Code switch indicators are used.
- The Nemeth Code switch indicators and math material are kept on the same braille line when possible.

Worksheets That Contain An Example Problem

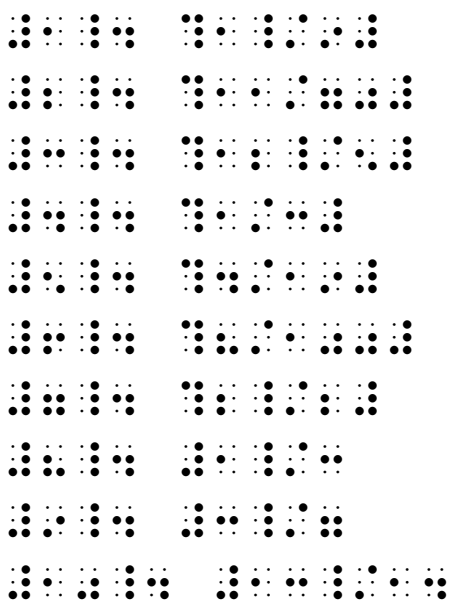
- Leave a blank line above and below the example (*Braille Formats*, 10.8.3).
- Transcribe the example problem like the other problems (*Braille Formats*, 10.8.4).
- When a colon follows the word “example” do not bold, underline, or italicize it (*Braille Formats*, 10.8.2).
- When no punctuation follows the word “example”, follow print formatting (*Braille Formats*, 10.8.1).

Chapter 5: Answer Key

Answer 5.1

1. $5/8$
2. $\frac{3}{16}$
3. $1/15$
4. $21/100$
5. $\frac{35}{100} = \_\_\_\_$
6. $\frac{3}{4} = \frac{9}{12}$
7. $\frac{1}{3} + \frac{1}{3} = ?$
8. $\frac{8}{20} < \frac{3}{7}$
9. $5/6 - 1/6 = \_\_\_\_$
10. $\frac{1}{3} \times \frac{9}{16} = \frac{1 \times 9}{3 \times 16}$
11. $? + 1/2 = 5/7$
12. $5/11 + 3/13 = \_\_\_\_$
13. $\frac{1}{100} = .01$

Answer 5.1 (continued)



Answer 5.2

1. $3\frac{1}{2}$

2. $12\ ^{5}\!/_{8}$

3. $5\ ^{6}\!/_{10}$

4. $384\ ^{2}\!/_{11}$

5. $1\frac{3}{5}$

6. $1\frac{1}{3} - \frac{1}{2}$

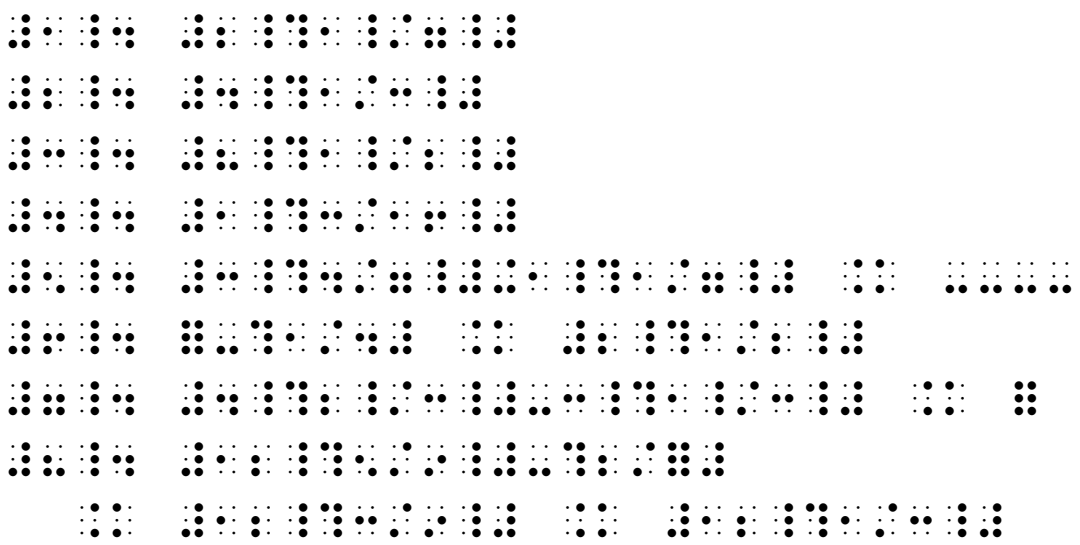
7. $15\frac{1}{4} + 7\frac{3}{8}$

8. $3 + 2\ ^{9}\!/_{10}$

9. $1{,}563\frac{3}{7} - 582\frac{1}{3}$

10. $\frac{1}{2} + 1\frac{2}{3}$

Answer 5.2 (continued)



Answer 5.3

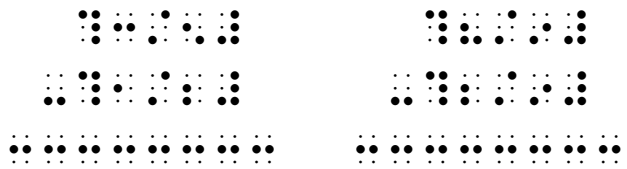
1. $\begin{array}{r} \frac{7}{12} \\ +\frac{3}{12} \\ \hline \end{array}$

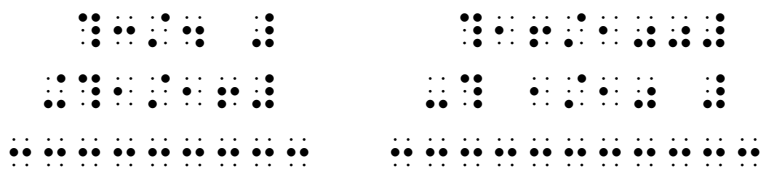
2. $\begin{array}{r} \frac{11}{15} \\ -\frac{6}{15} \\ \hline \end{array}$

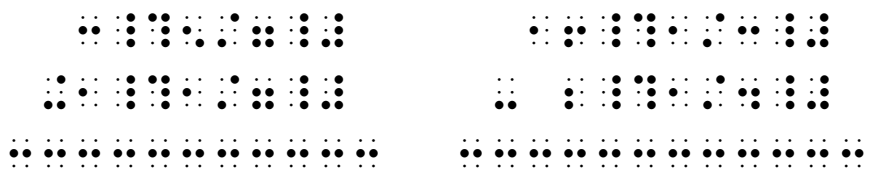
3. $\begin{array}{r} 8\frac{4}{24} \\ -3\frac{11}{6} \\ \hline \end{array}$

4. $\begin{array}{r} 8\ 17/20 \\ +14\ 3/5 \\ \hline \end{array}$

Answer 5.3 (continued)







Answer 5.4