

Chapter 6: Symbols Used in the Middle Grades

New Symbols Introduced

- ⋮ [dots 1-2-3-5-6] Left (open) parenthesis (()
- ⋮ [dots 2-3-4-5-6] Right (close) parenthesis ())
- ⋮ ⋮ [dot 4, dots 1-2-3-5-6] Left (open) bracket ([)
- ⋮ ⋮ [dot 4, dots 2-3-4-5-6] Right (close) bracket (])
- ⋮ ⋮ [dots 4-6, dots 1-2-3-5-6] Left (open) brace ({)
- ⋮ ⋮ [dots 4-6, dots 2-3-4-5-6] Right (close) brace (})
- ⋮ [dots 3-6] Negative sign or minus sign (−)
- ⋮ [dots 1-2-5-6] Vertical bar used with absolute value (| |)
- ⋮ ⋮ ⋮ [dot 5, dots 1-3, dots 1-5-6] Less than or equal to sign (\leq)
- ⋮ ⋮ ⋮ [dots 4-6, dot 2, dots 1-5-6] Greater than or equal to sign (\geq)
- ⋮ ⋮ ⋮ [dots 3-4, dots 4-6, dots 1-3] Not equal to sign (\neq)
- ⋮ ⋮ ⋮ ⋮ [dot 4, dots 1-5-6, dot 4, dots 1-5-6] Approximately equal to (\approx)
- ⋮ [dots 4-5] Superscript indicator (no print equivalent)
- ⋮ [dot 5] Baseline indicator (no print equivalent)
- ⋮ ⋮ [letter followed by a numeric subscript] Numeric subscript without indicators such as in the trapezoid area formula or in coordinate or ordered pairs (x_1)
- ⋮ [dots 3-4-5] Radical symbol $\sqrt{\quad}$
- ⋮ [dots 1-2-4-5-6] Termination indicator (no print equivalent)

Number line symbols

- ⠨ [dots 2-5] Line (axis line)
- ⠠ [dots 1-2-3-5] Coordinate scale mark
- ⠠ [dots 2-4-6] Left-pointing arrowhead
- ⠡ [dots 1-3-5] Right-pointing arrowhead
- ⠠ [dots 1-2-3-4-5-6] Solid, filled-in, or closed circle (point included) placed above the number line
- ⠡ [dots 1-3-4-6] Open circle (point not included) placed above the number line
- ⠠ [dots 2-3-5-6] Bold shaded line segment
- ⠠⠠ [dots 2-4-6 twice] Bold left-pointing arrowhead
- ⠡⠡ [dots 1-3-5 twice] Bold right-pointing arrowhead

New BANA Terms

- **Grouping symbols:** Parentheses, brackets, and braces are sometimes called grouping symbols.
- **Superscripts:** Exponents and other raised symbols such as the degree sign are called superscripts in Nemeth Code.
- **Baseline indicator:** Symbols that follow a baseline indicator are on the baseline.
- **Subscripts:** Symbols that are written below baseline are called subscripts, such as the "2" in H₂O.

Key Points

- **Variables** are letters that represent unknown numbers.
- Do not use an English letter indicator with a variable that is in a math expression.
- Do not italicize variables in braille.
- **Parentheses** are **grouping symbols** that usually have a beginning and an ending, often called an opening and a closing parenthesis.
- **Brackets** are also **grouping symbols** that are used as a second level of parentheses.

- **Braces** are also a type of **grouping symbol**. They are used as a third level of parentheses. They are also sometimes called curly brackets.
- **Absolute value** is represented by a vertical bar on each side of a number or expression.
- With a few exceptions such as numbers in matrices and equations in enlarged braces in Algebra, the numeric indicator is not used within grouping symbols.
- Do not use the English letter indicator when a single letter is enclosed in grouping symbols.
- Sometimes a closing parenthesis follows a problem number. When this occurs, the numeric indicator is used.
- **Negative numbers** are numbers that are less than zero. The negative sign is placed before the numeric indicator when writing a negative number.
- In a linear problem, **signs of comparison** (not equal to, less than or equal to, greater than or equal to, and approximately equal to) have a space before and after them.
- When a **superscript** is immediately followed by a mathematical symbol, the **baseline indicator** must be used to end the superscript and return to baseline.
- No baseline indicator is needed if a space follows a superscript.
- Do not use a **subscript indicator** or a **baseline indicator** when a variable has a numeric subscript.
- The **radical symbol** (dots 3-4-5) begins a square root and the **termination indicator** (dots 1-2-4-5-6) ends or terminates the square root.
- There are specific formatting rules and symbols for transcribing **number line graphs**, including inequalities, in Nemeth Code.

Introduction

Students are usually introduced to a variety of new symbols during the middle grades. In addition, previously learned concepts are expanded. New symbols include variables, grouping symbols, negative numbers, ordered pairs, absolute value, additional signs of comparison, superscripts including exponents, subscripts, and square roots.

Variables

Students are introduced to variables in sixth grade (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015).

Education, 2015). There are several types of grouping symbols including parentheses, brackets, braces, and vertical bars.

It takes one cell to write the **opening (left) parenthesis** in Nemeth Code. It is written with dots 1-2-3-5-6.

(
⋮

Teaching Tip: It may be helpful to point out that the opening parenthesis contains the same dots as the “of” groupsign.

It takes one cell to write the **closing (right) parenthesis** in Nemeth Code. It is written with dots 2-3-4-5-6.

)
⋮

Teaching Tip: It may be helpful to point out that the closing parenthesis contains the same dots as the “with” groupsign.

Teaching Tip: Remind students to “close” what they “open”. Thus, if they begin with an opening parenthesis, they will typically need a closing parenthesis at the end of the expression.

When a single number or mathematical expression is enclosed within grouping symbols, the numeric indicator is not usually used. You will learn about these exceptions in Chapter 8. When a single letter is enclosed within grouping symbols, the English letter indicator is not used.

Example 6.2 contains five problems that use parentheses. Notice that the problem identifiers are capital letters in parentheses.

Example 6.2

(A) $8 + (12 \div 4) = 11$

(B) $10.5 - (1.25 + 3.75) = ?$

(C) $(\$2.50 + \$4.75) \times 3 < \$25.00$

(D) $2(x + 3) = 2x + 6$

(E) Use substitution to evaluate $3y - 4z$ when $y = 5$ and $z = 1$.

Use your braillewriter to write the problems that contain variables and parentheses.

1. What is the value of d in $4(d + 5) = 5$?

2. Solve for p : $2p = 3 + 7$

3. $4x - 12 = 8$

4. $(3 + 3) \cdot 2 = 12$

5. $16 - t = 11$

Grouping Symbols: Brackets and Braces

Brackets are also **grouping symbols**. They are used as a second level of parentheses. It takes two cells to write a bracket in Nemeth Code. The opening (left) bracket is a dot 4, followed by the dots 1-2-3-5-6.

[

⠠⠠

The closing (right) bracket is a dot 4, followed by the dots 2-3-4-5-6.

]

⠠⠠

Braces are also a type of **grouping symbol**. They are used as a third level of parentheses. They are also sometimes called **curly brackets**. It takes two cells to write a brace in Nemeth Code. The opening brace is dots 4-6, followed by dots 1-2-3-5-6.

{

⠠⠠

The closing brace is dots 4-6, followed by dots 2-3-4-5-6.

}

⠠⠠

The numeric indicator is not used inside brackets or braces. Example 6.4 contains three problems containing brackets and/or braces.

Use your braillewriter to write the problems that include negative numbers, ordered pairs, and absolute value.

(a) $|-14| + |7| = \underline{\hspace{2cm}}$

(b) $|15| + |-6| = ?$

(c) $(-9, -5)$

(d) Is $-|-8| + |-3|$ positive or negative?

Introduction to Additional Signs of Comparison

During the middle grades, students are introduced to additional signs of comparison. For example, students learn to write inequalities that include a less than or equal to sign or a greater than or equal to sign in sixth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

Less Than or Equal to Sign

The **less than or equal to sign** is written with the less than sign followed by the horizontal bar. It takes three cells to write the less than or equal to sign in Nemeth Code. It is written with dot 5, followed by dots 1-3, and then dots 1-5-6.

\leq

⠠⠨⠠⠨⠠⠨

The less than or equal to sign is a **sign of comparison**, so there is a space before and after the symbol in braille (Rule XX). Example 6.8 contains three examples of inequalities that contain a less than or equal to sign.

Notice that the third example is a multiple choice problem. The problem begins in cell 1 with runover in cell 5. The answer choices begin in cell 3. Although all of the answer choices fit on one braille line, any runover would begin in cell 5.

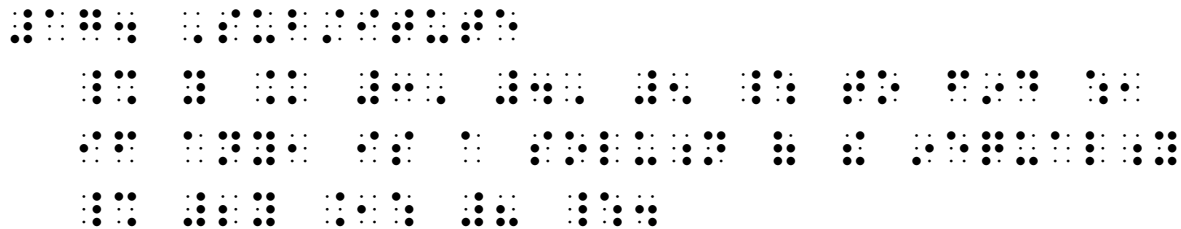
Example 6.8

$x \leq 3$

⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨

$y \leq -1$

⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨



Not Equal to Sign

Students are also introduced to the **not equal to sign** in sixth grade (Maryland Department of Education, 2015). It takes three cells to write the not equal to sign in Nemeth Code. It is written with dots 3-4, followed by dots 4-6, and then dots 1-3.

≠



Example 6.10 contains two examples with a not equal to sign. Also notice the negative numbers and absolute value in Example 6.10.

Example 6.10

$$|-37| \neq -37$$



$$|5 - 9| \neq -|9 - 5|$$



Approximately Equal to Sign

Students are also introduced to the **approximately equal to sign** in sixth grade (Maryland Department of Education, 2015). The approximately equal to sign is represented in print as parallel wavy lines. Each wavy line is called a tilde. It takes four cells to write the approximately equal to sign in Nemeth Code. It is written by dot 4, followed by dots 1-5-6, another dot 4, and then dots 1-5-6. The approximately equal to symbol has a space before and after it since it is a sign of comparison (Rule XX).

≈



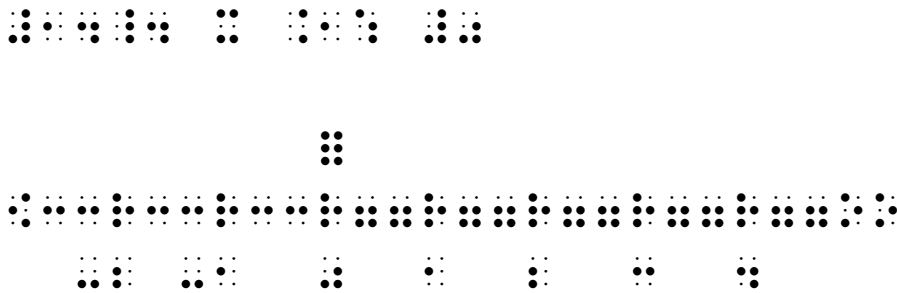
The **bold right pointing arrowhead**, which is placed on the right side of the number line, takes two cells to write in Nemeth Code. It is written with dots 1-3-5, followed by dots 1-3-5.



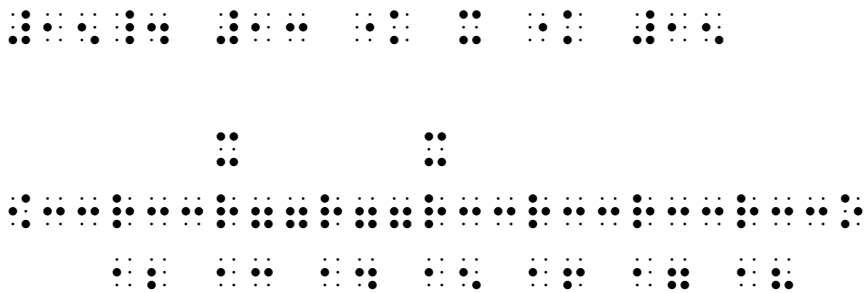
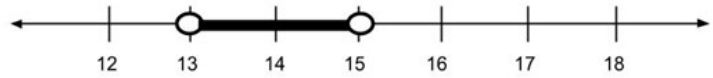
Example 6.12 includes four examples of graphing inequalities on a number line. Note the use of the bold shaded line segment in addition to the bold left and right pointing arrowheads. Both solid (filled-in) and open circles can be used when graphing inequalities.

Example 6.12

14. $x \geq 0$



15. $13 < x < 15$



$\sqrt{x^4}$



What is the value of $\sqrt{100}$?

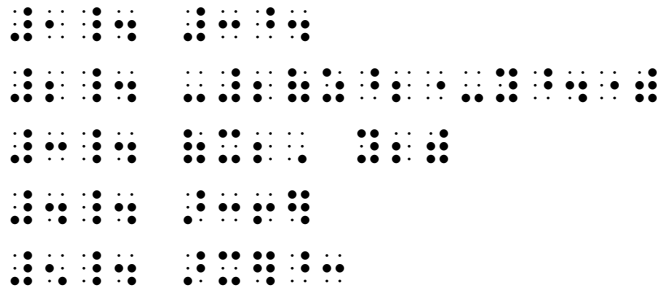


$\sqrt{x^4}$



Practice 6.5

Interline the following problems that contain superscripts, subscripts, and square roots.



Use your braillewriter to write the following problems that contain superscripts, subscripts, and square roots.

- c^5
- $6(z^4 - 3^2)$
- $A = \frac{(b_1 + b_2)}{2} h$
- $\sqrt{y^6}$

Chapter Summary

Variables

- Variables represent an unknown number.
- Write a variable in an equation or expression wherever it occurs.

Grouping Symbols

- Parentheses, brackets, and braces are grouping symbols (Rule XVIII).

- The vertical bar used with absolute value is also a grouping symbol (Rule XVIII, §124b).
- When a single number or mathematical expression is enclosed within grouping symbols, the numeric indicator is not usually used.
- When a single letter is enclosed within grouping symbols the English letter indicator is not used.

Negative Numbers

- The negative sign is placed before the numeric indicator when writing a negative number.

Ordered Pairs

- An ordered pair is a set of numbers used for plotting points on a coordinate plane.
- Ordered pairs are grouped in an opening parenthesis and a closing parenthesis with a mathematical comma (dot 6) between the horizontal (x) coordinate and the vertical (y) coordinate.
- Leave a space after the mathematical comma, even if there is not one in print.

Absolute Value

- Absolute value is represented by a vertical bar on each side of a number or expression.
- Do not use a numeric indicator within the vertical bars when writing an absolute value.

Less Than or Equal to Sign, Greater Than or Equal to Sign, Approximately Equal to Sign, and Not Equal to Sign

- These are signs of comparison.
- Leave a space before and after signs of comparison (Rule XX, §151).

Graphing Inequalities on a Number Line

- Students learn to interpret and graph number lines representing inequalities in the middle grades.
- Four additional Nemeth symbols are required to create these number lines.
 - The open circle, which represents a point that is not included, is written above the number line.
 - The bold (shaded) line segment is used for shading the rest of the points included in the solution on the number line.

- The bold left pointing arrowhead is placed on the left side of the number line.
- The bold right pointing arrowhead is placed on the right side of the number line.

Superscript Indicator and Baseline Indicator

- There are no print equivalents for the superscript and baseline indicators.
- The superscript indicator is used for raised symbols, including exponents and degree symbols.
- No baseline indicator is needed if a space follows a superscript.
- If anything comes after the exponent other than a space, a baseline indicator (dot 5) is used after the exponent.

Subscripts without Indicators

- Do not use a subscript indicator or a baseline indicator when a variable has a numeric subscript.

Square Roots

- The radical symbol begins a square root, and the termination indicator ends the square root.
- The numeric indicator is not used between the radical symbol and the termination indicator.
- There is no print equivalent for the termination indicator.

Chapter 6: Answer Key

Answer 6.1

1. $x(8 + 4) = 36$

2. $2(15 \div 3) = 10$

3. $(3 - 2 + 14) \times 8 = ?$

4. $(13 + 8) \div 3 = \underline{\quad}$

5. Use substitution to evaluate $5a + 4b$ when $a = 6$ and $b = 3$.

$x(8 + 4) = 36$
 $2(15 \div 3) = 10$
 $(3 - 2 + 14) \times 8 = ?$
 $(13 + 8) \div 3 = \underline{\quad}$
 Use substitution to evaluate $5a + 4b$ when $a = 6$ and $b = 3$.

Answer 6.2

21. Solve $(5 + m) - 6 = 4$

22. $6 \times [(x - 4) + 5] = 42$

23. $8 + \{[2(8 - 7) + 3]\} + 0 = ?$

24. $2 \times [3 + \frac{1}{5}(25 \div 5)] = ?$

$(5 + m) - 6 = 4$
 $6 \times [(x - 4) + 5] = 42$
 $8 + \{[2(8 - 7) + 3]\} + 0 = ?$
 $2 \times [3 + \frac{1}{5}(25 \div 5)] = ?$

Answer 6.4 (continued)

$$\frac{2x^2 + 3x - 2}{x^2 - 4} = \frac{2x^2 + 3x - 2}{(x-2)(x+2)}$$

$$= \frac{2x^2 + 3x - 2}{(x-2)(x+2)}$$

$$\frac{2x^2 + 3x - 2}{(x-2)(x+2)}$$

Answer 6.5

1. 3^4
2. $-2(z^2 - y^4)$
3. (x_2, y_2)
4. $\sqrt{36}$
5. $\sqrt{x^3}$

$$\frac{2x^2 + 3x - 2}{x^2 - 4} = \frac{2x^2 + 3x - 2}{(x-2)(x+2)}$$

$$= \frac{2x^2 + 3x - 2}{(x-2)(x+2)}$$