Chapter 7: Geometry for All Grade Levels

New Symbols Introduced

Shapes

⠫ [dots 1-2-4-6] Shape indicator (no print equivalent)

⠫⠉ [dots 1-2-4-6, dots 1-4] Circle (○)

⠫⠲ [dots 1-2-4-6, dots 2-5-6] Square (□)

⠫⠛ [dots 1-2-4-6, dots 2-5-6] Parallelogram (▱)

⠫⠞ [dots 1-2-4-6, dots 2-3-4-5] Triangle (△)

⠫⠗ [dots 1-2-4-6, dots 1-2-3-5] Rectangle (▭)

⠫⠙ [dots 1-2-4-6, dots 1-4-5] Diamond (◇)

⠫⠪ [dots 1-2-4-6, dots 2-4-6] Angle (∠)

⠫⠕ [dots 1-2-4-6, dots 1-3-5] Right arrow (contracted) (→)

Signs of comparison

⠈⠱ [dot 4, dots 1-5-6] Similar to (∼)

⠈⠱⠨⠅ [dot 4, dots 1-5-6, dots 4-6, dots 1-3] Congruent to (≅)

⠌⠈⠱⠨⠅ [dots 3-4, dot 4, dots 1-5-6, dots 4-6, dots 1-3]
Not congruent to (≇)

⠫⠇ [dots 1-2-4-6, dots 1-2-3] Parallel to (∥)

⠫⠏ [dots 1-2-4-6, dots 1-2-3-4] Perpendicular to (⊥)

⠌⠫⠇ [dots 3-4, dots 1-2-4-6, dots 1-2-3] Not parallel to (∦)

⠌⠫⠏ [dots 3-4, dots 1-2-4-6, dots 1-2-3-4] Not perpendicular to ($\not\perp$)

Other symbols

⠘⠨⠡ [dots 4-5, dots 4-6, dots 1-6] Degree sign (°)

⠄ [dot 3] Single prime (′)

⠄⠄ [dot 3, dot 3] Double prime (″)

⠨ [dots 4-6] Greek letter indicator (no print equivalent)

⠨⠹ [dots 4-6, dots 1-4-5-6] Pi (π)

Modified expressions

⠐ [dot 5] Multipurpose indicator (no print equivalent)

⠣ [dots 1-2-6] Directly over indicator (no print equivalent)

⠩ [dots 1-4-6] Directly under indicator (no print equivalent)

⠻ [dots 1-2-4-5-6] Termination indicator (no print equivalent)

New BANA Terms

- **Shape indicator**: Shapes, such as a square or triangle, are represented in braille using the shape indicator followed by a shape symbol – either a number or one or more letters suggestive of the name of the shape being represented.
- **Identifier**: When a shape is followed by some type of identification such as a letter, sequence of letters, or a number (e.g., $\angle 1$), the identification is called an identifier.
- **Greek letter indicator**: Letters of the Greek alphabet begin with dots 4-6 to identify it as a Greek letter.
- **Five-step rule**: The five-step rule is used when writing modified expressions such as $\overleftrightarrow{AB}$. It is called the five-step rule because there are five steps that must be used in the same order each time an expression is modified.
- **Multi-purpose indicator:** Dot 5 is used in a variety of ways in Nemeth Code to avoid confusion between symbols. It is also used before a modified expression to alert the reader of an upcoming modification.

Key Points

- **Shapes** are miniature pictures of geometric figures or other objects.
- Beginning in fourth grade, the **shape indicator** is used with a shape symbol (e.g., c for circle) when representing a shape. There is no print equivalent for the shape indicator. When a shape represents an **omission,** the shape is spaced the same way as the missing item which it represents.
- For students in Kindergarten through third grade, make a tactile graphic for the shape.
- When a shape represents an **omission,** the shape is spaced the same way as the missing item which it represents.
- **Regular polygons** (except for triangles), where all angles and all sides are equal, are written with a shape indicator followed by the number of sides of the shape.
- To avoid confusion, when a number follows a regular polygon (e.g., square) representing a missing sign of operation, a **multipurpose indicator** is placed between the numeric shape symbol and the following number.
- **Irregular polygons** and certain other shapes are written with a shape indicator followed by one or more letters or dot configurations suggestive of the name or look of the print shape being represented (e.g., r for rectangle and d for diamond).
- Place a space between the shape and its **identifier** (letter, sequence of letters, or a number that follows it), even if there is not a space in print.
- The **English letter indicator** is not used when writing a shape.
- A variety of **signs of comparison** (e.g., congruent to, parallel to) are used in geometry. Leave a space before and after a sign of comparison.
- The **degree sign** is a superscript.
- When writing **"the measure of" an angle** use "m" without an English letter indicator. If the "m" is italicized in print, do not italicize the "m" in braille.
- Use a **capitalization indicator** in front of each letter when brailling a sequence of letters such as in $\triangle ABC$.
- The **single prime** and **double prime** signs are used for feet and inches and for minutes and seconds. There is no space between the single or double prime signs and the quantities to which they apply.
- When writing **transformations**, include the right arrow (contracted) and read it as "maps to". Use prime notation along with the arrow to map a

pre-image to a new transformed image (single prime) or even another transformed image (double prime).
- Use the **Greek letter indicator** when brailling the lowercase Greek letter **pi**.
- Some of the Nemeth Code symbols used with modified expressions include: the **directly over indicator**, **directly under indicator**, **horizontal bar**, **two-way horizontal arrow**, and **right arrow (contracted)**.
- When brailling **modified expressions** follow the **five-step rule**.

Introduction

Students are first introduced to geometry in Kindergarten beginning with basic shapes such as circle and square (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015). For students in Kindergarten through third grade, use manipulatives or make a tactile graphic for two-dimensional shapes. For three-dimensional shapes such as a cube or a cone, only use 3D manipulatives. General education teachers likely have manipulatives that can be used with students with visual impairments.

Students are introduced to additional geometry concepts in fourth through eighth grade (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015). During high school, most students complete a geometry course (Common Core State Standards Initiative, 2010).

Shapes

Beginning in fourth grade, the **shape indicator** is used with a shape symbol (e.g., t for triangle) when representing a shape. It takes one cell to write the shape indicator in Nemeth Code. It is written with dots 1-2-4-6.

⠫

There is no print equivalent to the shape indicator. It is paired with the symbol representing the shape. Regular polygons (except for triangles), where all angles and all sides are equal, are written with the shape indicator followed by the number of sides of the shape (Rule XVI, §106 and §109). Notice that a numeric indicator is not used.

⠫⠲ □ Square

⠫⠢ ⬠ Pentagon

⠫⠖ ⬡ Hexagon

Irregular polygons and certain other shapes are written with the shape indicator followed by the symbol that represents the shape. The shape symbol is composed of one or more letters or dot configurations suggestive of the name or look of the print shape being represented (Rule XVI, §106 and §107).

⠫⠉ ○ Circle

⠫⠛ ▱ Parallelogram

⠫⠞ △ Triangle

⠫⠗ ▭ Rectangle

⠫⠙ ◇ Diamond

⠫⠪ ∠ Angle

⠫⠕ → Right arrow (contracted)

When brailling a shape, do not put a space between the shape indicator and the symbol that represents the shape.

However, put a space between the shape symbol and its identifier (e.g., the number or letters(s) that follow it) even if it does not look like there is one in print (Rule XVI, §115a). For instance in $\triangle ABC$ (read as triangle ABC), there is a space between the triangle and the letters that follow it in braille. Each letter must be individually capitalized, and italics are not used (BANA Guidance, 2018, p. 8). However, do not read the letters as capitalized letters.

Example 7.1 contains five examples of shapes. Notice that in the second example there does not appear to be a space between the circle symbol and the letter S in print. However, a space has been used in braille. In the examples, the letters are italicized in print, but they are not italicized in braille.

Example 7.1

$\angle B$

⠫⠪ ⠠⠃

$\bigcirc S$

⠫⠉ ⠠⠎

△*ABC*

⠫⠞ ⠠⠁⠠⠃⠠⠉

∠1

⠫⠪ ⠼⠂

□*EFGH*

⠫⠲ ⠠⠑⠠⠋⠠⠛⠠⠓

Signs of Comparison Used in Geometry

Students are first introduced to the following symbols in seventh or eighth grade: similar to, congruent to, and not congruent to (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015). These symbols are signs of comparison.

It takes two cells to write the **similar to** symbol in Nemeth Code. In print, it is sometimes called a tilde. It is written with dot 4, followed by dots 1-5-6.

∼

⠈⠱

It takes four cells to write the **congruent to** symbol in Nemeth Code. The congruent to symbol is a combination of the tilde and the equals sign. It is written with dot 4, followed by dots 1-5-6, then dots 4-6, followed by dots 1-3.

≅

⠈⠱⠨⠅

It takes five cells to write the **not congruent to** symbol in Nemeth Code. It is written with dots 3-4, followed by dot 4, followed by dots 1-5-6, followed by dots 4-6, followed by dots 1-3.

≇

⠌⠈⠱⠨⠅

> **Teaching Tip:** Comparison signs can be negated by placing dot 3-4 before the symbol in Nemeth Code. Therefore, the is not congruent to symbol and the not equal to symbol both begin with dots 3-4.

Example 7.2 contains three examples showing these new signs of comparison used in geometry. Notice there is a space before and after signs of comparison.

Example 7.2

$\angle 1 \cong \angle 2$

⠫⠪ ⠼⠂ ⠈⠱⠨⠅ ⠫⠪ ⠼⠆

$\angle A \ncong \angle B$

⠫⠪ ⠠⠁ ⠌⠈⠱⠨⠅ ⠫⠪ ⠠⠃

$\square ABCD \sim \square WXYZ$

⠫⠻ ⠠⠁⠠⠃⠠⠉⠠⠙ ⠈⠱ ⠫⠻ ⠠⠺⠠⠭⠠⠽⠠⠵

Practice 7.1

Interline the following problems that contain shapes.

⠼⠂⠸⠲ ⠫⠉ ⠠⠟

⠼⠆⠸⠲ ⠫⠪ ⠼⠂ ⠌⠈⠱⠨⠅ ⠫⠪ ⠼⠒

⠼⠒⠸⠲ ⠫⠲ ⠠⠁⠠⠃⠠⠉⠠⠙ ⠈⠱ ⠫⠲ ⠠⠑⠠⠋⠠⠛⠠⠓

⠼⠲⠸⠲ ⠫⠞ ⠠⠇⠠⠍⠠⠝ ⠈⠱⠨⠅ ⠫⠞ ⠠⠏⠠⠟⠠⠗

Use your braillewriter to write the following problems that contain shapes.

1. $\bigcirc Q$
2. $\triangle DEF \sim \triangle XYZ$
3. $\square PQRS \cong \square ABCD$
4. $\angle 2 \ncong \angle 7$

The Measure of an Angle

The letter m in front of an angle notation means "**the measure of**." The English letter indicator is not used with the letter m. Also do not italicize the letter m, even if it appears in italics in print.

The **degree sign** is often used when writing the measure of an angle. As previously covered in Chapters 3 and 6, it takes three cells to write the **degree sign** in Nemeth Code. It is written with dots 4-5, followed by dots 4-6, followed by dots 1-6.

$^\circ$

⠘⠨⠡

> **Teaching Tip:** $\angle ABC$ (read as angle ABC) denotes the angle itself, whereas $\mathrm{m}\angle ABC$ (read as the measure of angle ABC) is its measure. Angles can be congruent to each other, and the measures of angles can be equal to each other.

Example 7.3 contains three examples that include a measure of an angle. Notice that the degree sign is not used in the second example.

Example 7.3

$m\angle 1 = 60^\circ$

⠍⠫⠪ ⠼⠂ ⠨⠅ ⠼⠖⠴⠘⠨⠡

$m\angle 3 = m\angle 5$

⠍⠫⠪ ⠼⠒ ⠨⠅ ⠍⠫⠪ ⠼⠢

$m\angle XYZ = 120^\circ$

⠍⠫⠪ ⠠⠭⠠⠽⠠⠵ ⠨⠅ ⠼⠂⠆⠴⠘⠨⠡

Using Signs of Operation with Measures of Angles

Do not put a space between measures of angles and a sign of operation.

Example 7.4 provides two examples of measures of angles used with signs of operation.

Example 7.4

$m\angle 3 - m\angle 4 = 30^\circ$

⠍⠫⠪ ⠼⠒⠤⠍⠫⠪ ⠼⠲ ⠨⠅ ⠼⠒⠴⠘⠨⠡

$$\mathrm{m}\angle XWY + \mathrm{m}\angle YWZ = \mathrm{m}\angle XWZ$$

⠍⠫⠪ ⠠⠭⠠⠺⠠⠽⠬⠍⠫⠪ ⠠⠽⠠⠺⠠⠵ ⠨⠅ ⠍⠫⠪ ⠠⠭⠠⠺⠠⠵

Shapes That Represent Omission

As a reminder, when a shape is used to represent a sign of omission in Kindergarten through third grade instructional materials, the actual shape is produced for the braille reader. The shape can be produced using graphic art tape, felt, stickers etc. Shape symbols may be used in materials for students in fourth grade and above.

When using shapes to represent omission, space them the same way that the item they replaced would be spaced.

Example 7.5 contains four problems in which a shape is used to represent omission for students in fourth grade and above. Notice the square and rectangle represent a sign of comparison in the first and second problems. Therefore, there is a space on either side of the shape symbol. In the third example, the shape represents a missing number so there is no space between it and the multiplication sign. In the fourth example, the shape represents a sign of operation so there is no space between the triangle and the letters that precede and follow it.

Example 7.5

$$\angle 1 \ \square \ \angle 5$$

⠫⠪ ⠼⠂ ⠫⠲ ⠫⠪ ⠼⠢

$$\mathrm{m}\angle ABC \ \boxed{} \ 20^\circ$$

⠍⠫⠪ ⠠⠁⠠⠃⠠⠉ ⠫⠗ ⠼⠆⠴⠘⠨⠡

$$4 \times \bigcirc = 12$$

⠼⠲⠈⠡⠫⠉ ⠨⠅ ⠼⠂⠆

$$x \triangle y = xy$$

⠭⠫⠞⠽ ⠨⠅ ⠭⠽

Use of the Multipurpose Indicator with Regular Polygons

Regular polygons, such as the square, are represented by the shape indicator followed by a number. In the case of a square, the number is 4. To avoid confusion, when a number follows a regular polygon representing a missing sign of operation, a multipurpose indicator is placed between the shape symbol and the number (Rule XXIII, §177viii).

Examine the problem $92 \square 8 = 100$ written incorrectly in braille as:

⠼⠔⠆⠫⠲⠦ ⠨⠅ ⠼⠂⠴⠴

If a multipurpose indicator is not used, the braille reader will read this problem as "92 regular polygon with 48 sides equals 100." Although there is such a thing as a 48-gon, that is definitely not what the math teacher intended students to read! By placing the multipurpose indicator between the square and the 8, the braille reader is alerted to the shape ending and the number beginning, in this case the number 8.

Examine the problem transcribed correctly below and note the multipurpose indicator has been placed between 4 and the 8.

⠼⠔⠆⠫⠲⠐⠦ ⠨⠅ ⠼⠂⠴⠴

The multipurpose indicator is only used with a regular polygon when the item following the regular polygon, representing a missing sign of operation, is a number.

Example 7.6 contains four problems. Notice in the first and second problems, the regular polygons (squares) are followed by numbers so the multipurpose indicator is used. In the third problem, the regular polygon is not followed by a number so the multipurpose indicator is not used. In the final problem, the multipurpose indicator is used because the hexagon (a 6-sided shape) is followed by a number.

Example 7.6

$x + 4 \square 3 = 15$

⠭⠬⠲⠫⠲⠐⠒ ⠨⠅ ⠼⠂⠢

$20 = 80 \square 4$

⠼⠆⠴ ⠨⠅ ⠼⠦⠴⠫⠲⠐⠲

$$\frac{10}{50} = \frac{1}{\square}$$

⠹⠂⠴⠌⠢⠴⠼ ⠨⠅ ⠹⠂⠌⠫⠲⠼

$$24 \hexagon 4 = 6$$

⠼⠆⠲⠫⠖⠐⠲ ⠨⠅ ⠼⠖

Practice 7.2

Interline the following problems that contain shapes.

⠼⠂⠸⠲ ⠍⠫⠪⠠⠙⠠⠑⠠⠋ ⠨⠅ ⠼⠲⠢⠘⠨⠡
⠼⠆⠸⠲ ⠼⠖⠢ ⠨⠂⠱ ⠼⠦⠲⠫⠲⠐⠆
⠼⠒⠸⠲ ⠍⠫⠪ ⠼⠖⠤⠍⠫⠪⠼⠒ ⠨⠅ ⠼⠔⠴⠘⠨⠡
⠼⠲⠸⠲ ⠹⠫⠲⠌⠒⠼ ⠨⠅ ⠹⠲⠌⠖⠼

Use your braillewriter to write the following problems that contain shapes.

1. $m\angle LMN + m\angle PQR = 63^\circ$
2. $50\square 5 \neq 100\triangle 10$
3. $\mathrm{m}\angle 8 = 42^\circ$
4. $\frac{25}{5} = \frac{\square}{6}$

Signs of Shape Surrounded by Text

Signs of shape are mathematical; therefore, they must be enclosed in Nemeth Code switch indicators. Remember, keep the opening Nemeth Code indicator, the math content, and the Nemeth Code terminator on the same line if possible (BANA Guidance, 2018, p. 4).

A shape symbol and its identifier are considered a single item in Nemeth Code, so do not divide the shape indicator symbol and its identifier between lines.

Example 7.7 contains two word problems in which shapes are surrounded by text. In the first word problem, the shape symbol and its identifier are placed on the second line of braille. In the second word problem, the math content is divided across lines since it would not fit on a single line. It is permissible to have the Nemeth terminator and following punctuation on a line by themselves.

Example 7.7

1. Name the parallel sides of $\square ABCD$.

⠼⠁⠲ ⠠⠐⠝ ⠮ ⠏⠜⠁⠇⠇⠑⠇ ⠎⠊⠙⠑⠎ ⠷
⠸⠩ ⠫⠗ ⠠⠁⠠⠃⠠⠉⠠⠙ ⠸⠱⠲

2. In $\square WXYZ$, where $m\angle W = 135°$, find $m\angle Z$.

⠼⠃⠲ ⠠⠔ ⠸⠩ ⠫⠛ ⠠⠺⠠⠭⠠⠽⠠⠵⠠ ⠠⠄⠐⠱
⠍⠫⠪ ⠠⠺ ⠨⠅ ⠼⠂⠒⠢⠘⠨⠡⠠ ⠠⠄⠋⠔⠙ ⠍⠫⠪ ⠠⠵
⠸⠱⠲

Practice 7.3

Interline the following problems that contain shapes surrounded by text.

⠼⠊⠲ ⠸⠩ ⠫⠞ ⠠⠁⠠⠃⠠⠉ ⠸⠱ ⠊⠎ ⠒⠛⠗⠥⠢⠞ ⠞⠕
⠸⠩ ⠫⠞ ⠠⠙⠠⠑⠠⠋ ⠸⠱⠲

⠼⠁⠚⠲ ⠠⠊⠋ ⠸⠩ ⠫⠪ ⠠⠁ ⠠⠄⠯ ⠫⠪ ⠠⠉ ⠸⠱ ⠜⠑
⠉⠕⠍⠏⠇⠑⠰⠞⠜⠽ ⠁⠝⠛⠇⠑⠎⠂ ⠮⠝
⠸⠩ ⠍⠫⠪ ⠠⠁⠬⠍⠫⠪ ⠠⠉ ⠨⠅ ⠼⠔⠴⠘⠨⠡ ⠸⠱⠲

Use your braillewriter to write the following directions and problems that contain shapes surrounded by text.

In the next problems, ☐ represents a sign of operation. Is it +, -, ×, or ÷ ? Fill in the box with the correct sign of operation.

1. If $\angle PAQ$ and $\angle QAR$ are a linear pair, then $m\angle PAQ \;\square\; m\angle QAR = 180°$.
2. If $\angle PAQ$ and $\angle QAR$ are a linear pair, then $180° \;\square\; m\angle QAR = m\angle PAQ$.

Distance Between Two Points

In geometry, when two capital letters are written side by side with no modification (e.g., AB), this represents the distance between point A and point B. Since distance is a measure, signs of operation can be used with distances, similar to using signs of operation with measures of angles.

For example,

$$AB + BC = AC$$

⠠⠁⠠⠃⠬⠠⠃⠠⠉ ⠨⠅ ⠠⠁⠠⠉

Each letter must be individually capitalized, and italics are not used. This problem is read “AB plus BC equals AC.” However, do not read the distances as capitalized letters.

> **Teaching Tip:** Point out to students that since each point has its own separate label, each letter needs its own capital letter indicator.

Example 7.8 contains three examples of distance between two points. Notice that the examples are not numbered and therefore are formatted as paragraphs with the first line beginning in cell 3 and runover lines beginning in cell 1.

Example 7.8

$$AC - AB = BC$$

⠠⠁⠠⠉⠤⠠⠁⠠⠃ ⠨⠅ ⠠⠃⠠⠉

If $PQ = QR$ and Q is between P and R, what is another name for point Q?

⠠⠊⠋ ⠸⠩ ⠠⠏⠠⠟ ⠨⠅ ⠠⠟⠠⠗ ⠸⠱ ⠯ ⠰⠠⠟ ⠊⠎ ⠆⠞
⠰⠠⠏ ⠯ ⠰⠠⠗⠂ ⠱⠁⠞ ⠊⠎ ⠁⠝⠕⠮⠗ ⠐⠝ ⠿ ⠏⠕⠔⠞
⠰⠠⠟⠦

If $AC = 5$, $BC = 9$, and B is between A and C, what is AB?

⠠⠊⠋ ⠸⠩ ⠠⠁⠠⠉ ⠨⠅ ⠼⠢⠠ ⠠⠃⠠⠉ ⠨⠅ ⠼⠔ ⠸⠱⠂ ⠯
⠰⠠⠃ ⠊⠎ ⠆⠞ ⠠⠁ ⠯ ⠰⠠⠉⠂ ⠱⠁⠞ ⠊⠎
⠸⠩ ⠠⠁⠠⠃ ⠸⠱⠦

Single and Double Prime Signs

Students begin measuring the lengths of objects as early as the second grade and begin measuring angles as early as fourth grade (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015). As they become more experienced with measuring, students are introduced to the single prime sign and double prime sign representing feet and inches or minutes and seconds.

It takes one cell to write the **single prime sign** in Nemeth Code. It is written with dot 3.

'

⠄

It takes two cells to write the **double prime sign** in Nemeth Code. It is written with dot 3, followed by dot 3.

"

⠄⠄

There is no space between a prime sign and the quantity to which it applies (Rule XXII, §172).

Example 7.9 contains four examples using the single and double prime signs. Notice that the first two examples use the prime signs to represent feet and inches while the third and fourth examples use the prime signs to represent minutes and seconds. Also, a baseline indicator (dot 5) is required after the degree sign in the third and fourth examples so that the number following the degree sign is on the baseline.

Example 7.9

6'4"

⠼⠖⠄⠲⠄⠄

$CD = 9'10''$

⠠⠉⠠⠙ ⠨⠅ ⠼⠔⠄⠂⠴⠄⠄

$36^\circ 30'15''$

⠼⠒⠖⠘⠨⠡⠐⠒⠴⠄⠂⠢⠄⠄

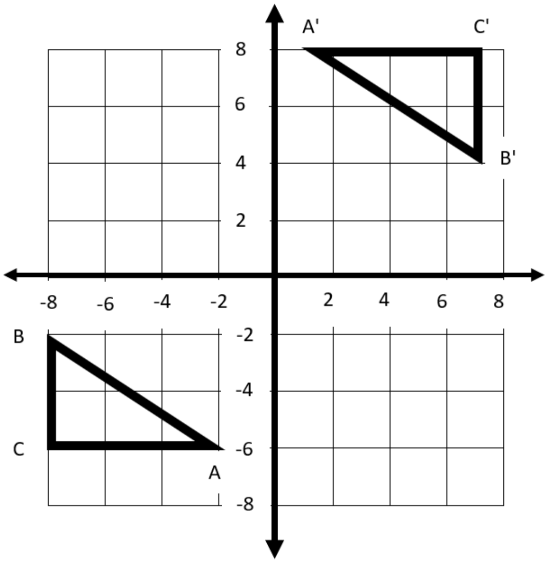
$\mathrm{m}\angle C = 44^\circ 10'35''$

⠍⠫⠪ ⠠⠉ ⠨⠅ ⠼⠲⠲⠘⠨⠡⠐⠂⠴⠄⠒⠢⠄⠄

Transformations

Students are introduced to **transformations** in eighth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

Geometric transformations involve taking a preimage and transforming it in some way to produce an image. In the following example, $\triangle ABC$ has been rotated 180° about the origin and then translated 2 units up and 1 unit to the left to create $\triangle A'B'C'$.



Prime notation and an arrow are used to map this pre-image to the new transformed image.

$$\triangle ABC \rightarrow \triangle A'B'C'$$

In Nemeth Code, the right arrow (contracted) is used to designate this mapping arrow. It takes two cells to write the **right arrow (contracted)** in Nemeth Code. It is written with dots 1-2-4-6, followed by dots 1-3-5.

$\rightarrow$

⠫⠕

When reading a transformation, the contracted right arrow is read as "maps to". Therefore, $\triangle ABC \rightarrow \triangle A'B'C'$ is read as "triangle ABC maps to triangle A prime B prime C prime," and it is brailled as:

⠫⠞ ⠠⠁⠠⠃⠠⠉ ⠫⠕ ⠫⠞ ⠠⠁⠄⠠⠃⠄⠠⠉⠄

Example 7.10 contains two examples of transformations. The second example is transcribed twice. Both transcriptions are correct since the *2018 Guidance for Transcription Using the Nemeth Code within UEB Contexts* provides some flexibility about where switch indicators should be placed. However, the second transcription saves a line in braille.

> **Teaching Tip:** It is important that students see multiple ways of transcribing the same thing, such as a transformation. Take time to familiarize students with different ways to present the same material so that if they come across different presentations on tests or in textbooks, they are able to read and understand them.

Example 7.10

$\triangle DEF \rightarrow \triangle D'E'F' \rightarrow \triangle D''E''F''$

⠫⠞ ⠠⠙⠠⠑⠠⠋ ⠫⠕ ⠫⠞ ⠠⠙⠄⠠⠑⠄⠠⠋⠄
⠫⠕ ⠫⠞ ⠠⠙⠄⠄⠠⠑⠄⠄⠠⠋⠄⠄

Draw a transformation of $\square ABCD \rightarrow \square A'B'C'D'$ that is a translation left 2".

⠠⠙⠗⠁⠺ ⠁ ⠞⠗⠁⠝⠎⠿⠍⠁⠰⠝ ⠷
⠸⠩ ⠫⠛ ⠠⠁⠠⠃⠠⠉⠠⠙ ⠫⠕ ⠫⠛ ⠠⠁⠄⠠⠃⠄⠠⠉⠄⠠⠙⠄ ⠸⠱
⠞ ⠊⠎ ⠁ ⠞⠗⠁⠝⠎⠇⠁⠰⠝ ⠇⠑⠋⠞
⠸⠩ ⠼⠆⠄⠄ ⠸⠱⠲ or

⠠⠙⠗⠁⠺ ⠁ ⠞⠗⠁⠝⠎⠿⠍⠁⠰⠝ ⠷ ⠸⠩
⠫⠛ ⠠⠁⠠⠃⠠⠉⠠⠙ ⠫⠕ ⠫⠛ ⠠⠁⠄⠠⠃⠄⠠⠉⠄⠠⠙⠄ ⠸⠱ ⠞
⠊⠎ ⠁ ⠞⠗⠁⠝⠎⠇⠁⠰⠝ ⠇⠑⠋⠞ ⠸⠩ ⠼⠆⠄⠄ ⠸⠱⠲

Pi

Students are introduced to the lowercase Greek letter pi in seventh grade, as they solve problems involving circles, the area of a circle, and the volume of a sphere (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

When writing any Greek letter, begin with the Greek letter indicator. It is written with dots 4-6. There is no print equivalent for the Greek letter indicator.

⠨

The lowercase Greek letter **pi**, is written with dots 4-6, followed by the letter "p" which is dots 1-2-3-4. Pi is a popular constant and is the ratio of the circumference of any circle to the diameter of that circle.

π

⠨⠏

Pi is used in geometry formulas such as:

$C = \pi d$

⠠⠉ ⠨⠅ ⠨⠏⠙

$A = \pi r^2$

⠠⠁ ⠨⠅ ⠨⠏⠗⠘⠆

$V = \frac{4}{3}\pi r^3$

⠠⠧ ⠨⠅ ⠹⠲⠌⠒⠼⠨⠏⠗⠘⠒

Example 7.11 contains two examples of word problems that use the Greek letter pi. Notice that the Nemeth Code switch indicators and math content are kept on the same line in both examples.

Example 7.11

1. If the radius of $\bigcirc P$ is 3.5, find the area of $\bigcirc P$, using the formula $A = \pi r^2$.

⠼⠁⠲ ⠠⠊⠋ ⠮ ⠗⠁⠙⠊⠥⠎ ⠷
⠸⠩ ⠫⠉ ⠠⠏ ⠠⠄⠊⠎ ⠼⠒⠨⠢ ⠸⠱⠂ ⠋⠔⠙ ⠮ ⠜⠑⠁ ⠷
⠸⠩ ⠫⠉ ⠠⠏ ⠸⠱⠂ ⠥⠎⠬ ⠮ ⠿⠍⠥⠇⠁
⠸⠩ ⠠⠁ ⠨⠅ ⠨⠏⠗⠘⠆ ⠸⠱⠲

2. Using the formula $V = \frac{4}{3}\pi r^3$, find the volume of a sphere if $r = 1'2''$.

⠼⠃⠲ ⠠⠥⠎⠬ ⠮ ⠿⠍⠥⠇⠁
⠸⠩ ⠠⠧ ⠨⠅ ⠹⠲⠌⠒⠼⠨⠏⠗⠘⠒ ⠸⠱⠂ ⠋⠔⠙ ⠮ ⠧⠕⠇⠥⠍⠑
⠷ ⠁ ⠎⠏⠐⠓ ⠊⠋ ⠸⠩ ⠗ ⠨⠅ ⠼⠂⠄⠆⠄⠄ ⠸⠱⠲

Practice 7.4

Interline the following problems that contain distance between points, prime signs, transformations, and pi.

⠼⠂⠸⠲ ⠠⠁⠠⠃ ⠨⠅ ⠼⠂⠒⠄⠖⠄⠄

⠼⠆⠸⠲ ⠍⠫⠪ ⠠⠙ ⠨⠅ ⠼⠲⠢⠘⠨⠡⠐⠂⠴⠄⠆⠢⠄⠄

⠼⠒⠸⠲ ⠫⠞ ⠠⠭⠠⠽⠠⠵ ⠫⠕ ⠫⠞ ⠠⠭⠄⠠⠽⠄⠠⠵⠄

⠀⠀⠫⠕ ⠫⠞ ⠠⠭⠄⠄⠠⠽⠄⠄⠠⠵⠄⠄

⠼⠲⠸⠲ ⠠⠉ ⠨⠅ ⠼⠆⠨⠏⠗

Use your braillewriter to write the problems that contain distance between points, prime signs, transformations, and the Greek letter pi.

1. $PQ = 8'2''$
2. $\square WXYZ \rightarrow \square W'X'Y'Z'$
3. $\text{m}\angle DEF = 34^\circ 52'14''$
4. $V = \frac{1}{3}\pi r^2 h$

Modified Expressions

Students are introduced to modified expressions as early as fourth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015). In print, a modified expression is formed when a symbol or combination of symbols are placed directly over or under its related symbol or expression.

Some common modifiers are:

- The horizontal bar, which can be placed over an expression or number to represent a line segment or repeating digits in a number (e.g., $\overline{AB}$ or $2.\overline{46}$)
- The barbed arrow at both ends to represent a line (e.g., $\overleftrightarrow{XY}$)
- The right pointing arrow to represent a ray (e.g., $\overrightarrow{CD}$)

When transcribing most modified expressions in Nemeth Code, the **five-step rule** is used (Rule XIV, §86). It is called the five-step rule because there are five steps that must be used in the same order each time an expression is modified using specified symbols (Rule XIV, §86). There is no print equivalent for the four modification indicators.

Five-Step Rule

Step 1: Multipurpose indicator (dot 5)

⠐

Step 2: Expression being modified

Step 3: Directly over indicator (dots 1-2-6)

⠣

or **Directly under indicator** (dots 1-4-6)

⠩

Step 4: Modifier

Step 5: Termination indicator (dots 1-2-4-5-6)

⠻

Common Modifiers in Geometry

The fourth step requires a modifier. Three commonly used modifiers in geometry are the horizontal bar, barbed arrow at both ends, and right arrow (contracted).

The **horizontal bar** is written with dots 1-5-6.

—

⠱

The **barbed arrow at both ends** is written using five cells. It is written with dots 1-2-4-6, dots 2-4-6, dots 2-5, dots 2-5, and dots 1-3-5. This symbol begins with the shape indicator, but notice how the remaining four cells visually look like the print line they are representing.

↔

⠫⠪⠒⠒⠕

As previously noted, the **right arrow (contracted)** is written using two cells. It is written with dots 1-2-4-6, followed by dots 1-3-5. This symbol also begins with the shape indicator, but notice that the next cell visually looks like a right-pointing arrowhead. In the example below, the right arrow (contracted) represents a ray.

$\rightarrow$

⠫⠕

Example 7.12 contains three modified expressions that might be used in geometry. Notice that each expression follows the five-step rule.

Example 7.12

$\overline{ST}$ (read as line segment ST)

⠐⠠⠎⠠⠞⠣⠱⠻

$\overleftrightarrow{PQ}$ (read as line PQ)

⠐⠠⠏⠠⠟⠣⠫⠪⠒⠒⠕⠻

$\overrightarrow{XY}$ (read as ray XY)

⠐⠠⠭⠠⠽⠣⠫⠕⠻

Modified Expressions Outside of Geometry

Modified expressions are used outside of geometry, including when transcribing multi-digit repeating decimals. If a horizontal bar is placed over two or more digits in a decimal number, the five-step rule must be followed.

Example 7.13 contains two examples of multi-digit repeating decimals.

Example 7.13

$20.48\overline{46}$ (read as twenty point four eight four six with the four six repeating)

⠼⠆⠴⠨⠲⠦⠐⠲⠖⠣⠱⠻

$0.\overline{317}$ (read as zero point three one seven with the three one seven repeating)

⠼⠴⠨⠐⠒⠂⠶⠣⠱⠻

Exceptions to the Five-Step Rule

If a horizontal bar is placed over a single digit in a decimal number, do not use the five-step rule. For expressions with only one repeating digit, there is a contracted form that makes them easier and quicker to read. Simply place the horizontal bar symbol immediately after the digit or letter it modifies (Rule XIV §86c).

For example, $0.\overline{3}$ is read as zero point three with the three repeating. It is transcribed:

⠼⠴⠨⠒⠱

Example 7.14 contains two examples showing a horizontal bar placed over one digit.

Example 7.14

$1.\overline{6}$ (read as one point six with the 6 repeating)

⠼⠂⠨⠖⠱

$0.74\overline{9}$ (read as zero point seven four nine with the 9 repeating)

⠼⠴⠨⠶⠲⠔⠱

Quick Review

When a horizontal bar is used directly under a single digit or letter, the five-step rule is not used as explained in Chapter 3 (BANA Nemeth Update: Rule XIV, § 86c Contracted Form of Underlining of Single Letters/Digits, 2013). Write the directly-under indicator (dots 1-4-6) and horizontal bar symbol (dots 1-5-6) immediately after the digit or letter.

To review, $72.\underline{4}$ is read as seventy-two point four with a bar under the 4. It is transcribed:

⠼⠶⠆⠨⠲⠩⠱

Since it is extremely rare to have a modified expression that uses the directly under indicator (other than place value), all exercise material will focus on modified expressions using the directly over indicator or the contracted form of the horizontal bar over a single digit.

Parallel, Perpendicular, Not Parallel, and Not Perpendicular

Students are introduced to parallel (sometimes called is parallel to) and perpendicular (sometimes called is perpendicular to) in fourth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

It takes two cells to write the **parallel to sign** in Nemeth Code. It is written with dots 1-2-4-6, followed by dots 1-2-3.

∥

⠫⠇

It takes two cells to write the **perpendicular to sign** in Nemeth Code. It is written with dots 1-2-4-6, followed by dots 1-2-3-4.

⊥

⠫⠏

Students are introduced to not parallel (sometimes called is not parallel to) and not perpendicular (sometimes called is not perpendicular to) in fourth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

As explained earlier in this chapter, comparison signs can be negated by placing dots 3-4 before the symbol in Nemeth Code.

It takes three cells to write the **not parallel to sign** in Nemeth Code. It is written with dots 3-4, followed by dots 1-2-4-6, followed by dots 1-2-3.

∦

⠌⠫⠇

It takes three cells to write the **not perpendicular sign** in Nemeth Code. It is written with dots 3-4, followed by dots 1-2-4-6, followed by dots 1-2-3-4.

⊥̸

⠌⠫⠏

Example 7.15 contains four examples that show the use of parallel, perpendicular, not parallel, and not perpendicular.

Example 7.15

$\overline{EF} \parallel \overline{GH}$

⠐⠠⠑⠠⠋⠣⠱⠻ ⠫⠇ ⠐⠠⠛⠠⠓⠣⠱⠻

$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠕⠻ ⠫⠏ ⠐⠠⠉⠠⠙⠣⠫⠪⠒⠒⠕⠻

$\overline{WX} \nparallel \overline{YZ}$

⠐⠠⠺⠠⠭⠣⠱⠻ ⠌⠫⠇ ⠐⠠⠽⠠⠵⠣⠱⠻

$\overrightarrow{PQ} \not\perp \overrightarrow{RS}$

⠐⠠⠏⠠⠟⠣⠫⠕⠻ ⠌⠫⠏ ⠐⠠⠗⠠⠎⠣⠫⠕⠻

Practice 7.5

Interline the following problems that contain modified expressions, parallel, perpendicular, not parallel, and not perpendicular.

⠼⠂⠸⠲ ⠼⠲⠨⠐⠂⠆⠒⠣⠱⠻

⠼⠆⠸⠲ ⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠕⠻

⠼⠒⠸⠲ ⠐⠠⠗⠠⠎⠣⠫⠕⠻

⠼⠲⠸⠲ ⠼⠂⠨⠖⠱

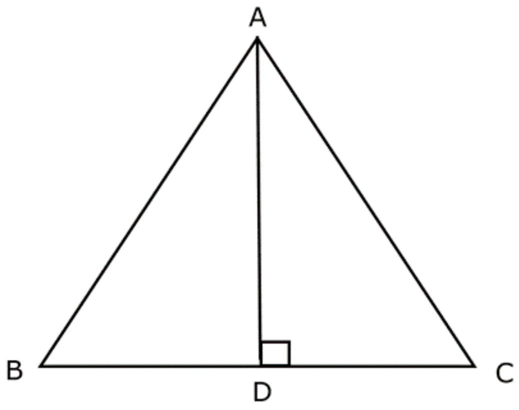
⠼⠢⠸⠲ ⠐⠠⠑⠠⠋⠣⠫⠕⠻ ⠌⠫⠏ ⠐⠠⠑⠠⠛⠣⠫⠕⠻

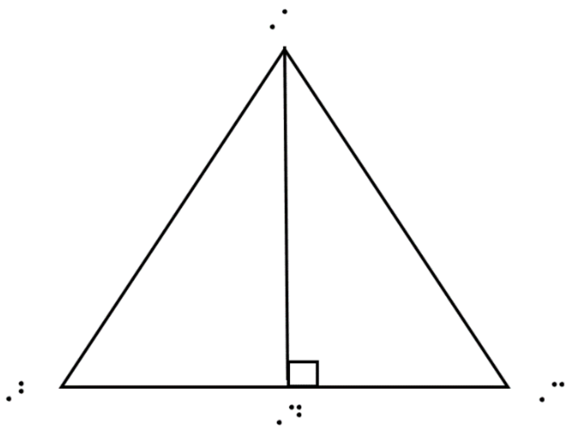
Use your braillewriter to write the following problems that contain modified expressions, parallel, perpendicular, not parallel, and not perpendicular.

1. If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, then $m\angle 4 = ?$
2. $\frac{11}{12} = 0.91\overline{6}$
3. If $\overleftrightarrow{PQ} \perp \overleftrightarrow{RS}$, then what is $m\angle ROQ$?
4. In $\square ABCD$, $\overline{AB} \nparallel \overline{BC}$.
5. $\frac{4}{11} = 0.\overline{36}$

Creating Simple Tactile Graphics

Triangles, circles, angles, and lines are often used in geometry problems. Mathematical diagrams aid all students in understanding concepts. These diagrams should be provided to braille readers as tactile graphics with braille labels. For example, the first triangle below was easily created as a black line master using the Microsoft Word Shapes and then labeled with the capital print letters using textboxes. The second triangle uses the same black line master and simply replaces the print letters with capital braille letters using textboxes and a braille font. This same graphic could have also been created using a braillewriter and graphic art tape.





Detailed information about how to create mathematical tactile graphics is available in Unit 6 of the *Guidelines and Standards for Tactile Graphics, 2010*. Additional tactile graphics are included in Part 1 of the End of the Chapter Exercise. Practice creating a simple tactile graphic is provided in Part 2.

Chapter Summary

Shapes

- The shape indicator is used with the assigned shape symbol to represent regular polygons and irregular shapes.
- The English letter indicator is not used when writing a shape.
- A space is left between the shape and its identifier, even if one does not appear in print.
- When a regular polygon (e.g., square) representing a missing sign of operation is followed by a number, a multipurpose indicator is placed between the numeric shape symbol and the following number.
- When a shape represents omitted material, space the shape the same way the omitted material would be spaced.

- When brailling the measure of an angle, use “m” before the angle symbol to represent “the measure of”. Do not use the English letter indicator or italicize the “m”.
- Use Nemeth Code switch indicators when shapes are surrounded by text.

Signs of Comparison Used in Geometry

- In geometry students are introduced to a variety of signs of comparison such as congruent to and parallel to.
- Place a space on either side of signs of comparison symbols.

Distance Between Two Points

- When writing the distance between two points (e.g., AB) place a capital indicator in front of each letter. Do not italicize the letters.

Single and Double Prime Signs

- The single prime sign is used for feet or minutes.
- The double prime sign is used for inches or seconds.
- There is no space between the single or double prime sign and the quantity to which it applies.

Transformations

- Transformations use the right arrow (contracted) as well as the single and/or double prime signs. The right arrow is read as “maps to”.

Pi

- The Greek letter indicator is used when writing the Greek letter pi.
- The letter “p” following the Greek letter indicator is used to represent pi.

Modified Expressions

- Common symbols used when creating modified expressions include the multipurpose indicator, directly over indicator, directly under indicator, horizontal bar, barbed arrow at both ends, right arrow (contracted), and the termination indicator.
- The five-step rule is followed when writing modified expressions such as line, line segment, ray, and multi-digit repeating decimals.
- There are two exceptions to the five-step rule – one for a single digit or letter with a horizontal bar written directly over it and another for a single digit or letter with a horizontal bar written directly under it.

Chapter 7: Answer Key

Answer 7.1

1. $\bigcirc P$
2. $\angle 1 \ncong \angle 3$
3. $\square ABCD \sim \square EFGH$
4. $\triangle LMN \cong \triangle PQR$

⠼⠂⠸⠲ ⠫⠉ ⠠⠟
⠼⠆⠸⠲ ⠫⠞ ⠠⠙⠠⠑⠠⠋ ⠈⠱ ⠫⠞ ⠠⠭⠠⠽⠠⠵
⠼⠒⠸⠲ ⠫⠲ ⠠⠏⠠⠟⠠⠗⠠⠎ ⠈⠱⠨⠅ ⠫⠲ ⠠⠁⠠⠃⠠⠉⠠⠙
⠼⠲⠸⠲ ⠫⠪ ⠼⠆ ⠌⠈⠱⠨⠅ ⠫⠪ ⠼⠶

Answer 7.2

1. $\text{m}\angle DEF = 45^\circ$
2. $65 \geq 84\square 2$
3. $\text{m}\angle 6 - \text{m}\angle 3 = 90^\circ$
4. $\frac{\square}{3} = \frac{4}{6}$

⠼⠂⠸⠲ ⠍⠫⠪ ⠠⠇⠠⠍⠠⠝⠬⠍⠫⠪ ⠠⠏⠠⠟⠠⠗
⠨⠅ ⠼⠖⠒⠘⠨⠡
⠼⠆⠸⠲ ⠼⠢⠴⠫⠲⠐⠢ ⠌⠨⠅ ⠼⠂⠴⠴⠫⠞⠂⠴
⠼⠒⠸⠲ ⠍⠫⠪ ⠼⠦ ⠨⠅ ⠼⠲⠆⠘⠨⠡
⠼⠲⠸⠲ ⠹⠆⠢⠌⠢⠼ ⠨⠅ ⠹⠫⠲⠌⠖⠼

Answer 7.3

9. $\triangle ABC$ is congruent to $\triangle DEF$.
10. If $\angle A$ and $\angle C$ are complementary angles, then $\text{m}\angle A + \text{m}\angle C = 90^\circ$.

Answer 7.3 (continued)

⠠⠔ ⠮ ⠝⠑⠭⠞ ⠏⠗⠕⠃⠇⠑⠍⠎⠂ ⠸⠩ ⠫⠲ ⠸⠱
⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠁ ⠎⠊⠛⠝ ⠯ ⠕⠏⠻⠁⠰⠝⠲ ⠠⠊⠎ ⠭
⠸⠩ ⠬⠠ ⠤⠠ ⠈⠡⠠ ⠠⠄⠕⠗ ⠨⠌ ⠸⠱⠦ ⠠⠋⠊⠇⠇ ⠔
⠮ ⠃⠕⠭ ⠾ ⠮ ⠉⠕⠗⠗⠑⠉⠞ ⠎⠊⠛⠝ ⠯ ⠕⠏⠻⠁⠰⠝⠲
⠼⠁⠲ ⠠⠊⠋
⠸⠩ ⠫⠪ ⠠⠏⠠⠁⠠⠟ ⠠⠄⠯ ⠫⠪ ⠠⠟⠠⠁⠠⠗ ⠸⠱ ⠜⠑
⠁ ⠇⠔⠑⠜ ⠏⠁⠊⠗⠂ ⠮⠝
⠸⠩ ⠍⠫⠪ ⠠⠏⠠⠁⠠⠟⠫⠲⠍⠫⠪ ⠠⠟⠠⠁⠠⠗
⠨⠅ ⠼⠂⠦⠴⠘⠨⠡ ⠸⠱⠲
⠼⠃⠲ ⠠⠊⠋
⠸⠩ ⠫⠪ ⠠⠏⠠⠁⠠⠟ ⠠⠄⠯ ⠫⠪ ⠠⠟⠠⠁⠠⠗ ⠸⠱ ⠜⠑
⠁ ⠇⠔⠑⠜ ⠏⠁⠊⠗⠂ ⠮⠝
⠸⠩ ⠼⠂⠦⠴⠘⠨⠡⠐⠫⠲⠍⠫⠪ ⠠⠟⠠⠁⠠⠗
⠨⠅ ⠍⠫⠪ ⠠⠏⠠⠁⠠⠟ ⠸⠱⠲

Answer 7.4

1. $AB = 13'6''$
2. $m\angle 4 = 45^\circ 10'25''$
3. $\triangle XYZ \rightarrow \triangle X'Y'Z' \rightarrow \triangle X''Y''Z''$
4. $C = 2\pi r$

⠼⠂⠸⠲ ⠠⠏⠠⠟ ⠨⠅ ⠼⠦⠄⠆⠄⠄
⠼⠆⠸⠲ ⠫⠲ ⠠⠺⠠⠭⠠⠽⠠⠵ ⠫⠕ ⠫⠲ ⠠⠺⠄⠠⠭⠄⠠⠽⠄⠠⠵⠄
⠼⠒⠸⠲ ⠍⠫⠪ ⠠⠙⠠⠑⠠⠋ ⠨⠅ ⠼⠒⠲⠘⠨⠡⠐⠢⠆⠄⠂⠲⠄⠄
⠼⠲⠸⠲ ⠠⠧ ⠨⠅ ⠹⠂⠌⠒⠼⠨⠏⠗⠘⠆⠐⠓

Answer 7.5

1. $4.\overline{123}$
2. $\overleftrightarrow{AB}$
3. $\overrightarrow{RS}$
4. $1.\overline{6}$
5. $\overrightarrow{EF} \not\perp \overrightarrow{EG}$

⠼⠂⠸⠲ ⠠⠄⠠⠊⠋ ⠐⠠⠁⠠⠃⠣⠫⠕⠻ ⠫⠇ ⠐⠠⠉⠠⠙⠣⠫⠕⠻⠠
⠠⠄⠮⠝ ⠍⠫⠪ ⠼⠲ ⠨⠅ ⠿
⠼⠆⠸⠲ ⠹⠂⠂⠌⠂⠆⠼ ⠨⠅ ⠼⠴⠨⠔⠂⠖⠱
⠼⠒⠸⠲ ⠠⠄⠠⠊⠋
⠐⠠⠏⠠⠟⠣⠫⠪⠒⠒⠕⠻ ⠫⠏ ⠐⠠⠗⠠⠎⠣⠫⠪⠒⠒⠕⠻ ⠸⠱
⠮⠝ ⠱⠁⠞ ⠊⠎ ⠸⠩ ⠍⠫⠪ ⠠⠗⠠⠕⠠⠟⠸⠦
⠼⠲⠸⠲ ⠠⠄⠠⠔ ⠫⠛ ⠠⠁⠠⠃⠠⠉⠠⠙⠠
⠐⠠⠁⠠⠃⠣⠱⠻ ⠌⠫⠇ ⠐⠠⠃⠠⠉⠣⠱⠻⠸⠲
⠼⠢⠸⠲ ⠹⠲⠌⠂⠂⠼ ⠸⠅ ⠼⠴⠨⠔⠒⠖⠣⠱⠻