Chapter 8: An Introduction to Nemeth Code Symbols Used in Algebra 1, Algebra 2, and Advanced Mathematics

New Symbols Introduced

⠳ [dots 1-2-5-6] Vertical bar used with such that, magnitude, norm, and determinants (|)

⠠⠿ [dots 1-2-3-4-5-6] Infinity symbol (∞)

⠨⠡ [dots 4-6, dots 1-6] Hollow dot (○)

⠘⠘ [dots 4-5, dots 4-5] Superscript with superscript indicator

⠰ [dots 5-6] Subscript indicator

⠣ [dots 1-2-6] Index-of-radical indicator

Grouping Symbols

⠨⠨⠷ [dots 4-6, dots 4-6, dots 1-2-3-5-6] Opening (left) angle bracket (⟨)

⠨⠨⠾ [dots 4-6, dots 4-6, dots 2-3-4-5-6] Closing (right) angle bracket (⟩)

Enlarged Grouping Symbols

⠠⠷ [dot 6, dots 1-2-3-5-6] Enlarged opening (left) parenthesis (( )

⠠⠾ [dot 6, dots 2-3-4-5-6] Enlarged closing (right) parenthesis ( ))

⠈⠠⠷ [dot 4, dot 6, dots 1-2-3-5-6] Enlarged opening (left) bracket ([ )

⠈⠠⠾ [dot 4, dot 6, dots 2-3-4-5-6] Enlarged closing (right) bracket ( ])

⠨⠠⠷ [dots 4-6, dot 6, dots 1-2-3-5-6] Enlarged opening (left) brace ({)

⠨⠠⠾ [dots 4-6, dot 6, dots 2-3-4-5-6] Enlarged closing (right) brace (})

⠠⠳ [dot 6, dots 1-2-5-6] Enlarged vertical bar (|)

Set Theory

⠸⠴ [dots 4-5-6, dots 3-5-6] Empty set (null set) (∅)

⠨⠬ [dots 4-6, dots 3-4-6] Union (∪)

⠨⠩ [dots 4-6, dots 1-4-6] Intersection (∩)

⠸⠐⠨⠱ [dots 4-5-6, dot 5, dots 4-6, dots 1-5-6 1] is a subset of (⊆)

⠈⠑ [dot 4, dots 1-5] Is an element of (∈)

Greek Symbols

⠨⠁ [dots 4-6, dot 1] Greek letter alpha (lowercase) (α)

⠨⠃ [dots 4-6, dots 1-2] Greek letter beta (lowercase) (β)

⠨⠹ [dots 4-6, dots 1-4-5-6] Greek letter theta (lowercase) (θ)

⠨⠁⠙ [dots 4-6, dot 1, dots 1-4-5] Greek letter delta (uppercase) (Δ)

⠨⠁⠎ [dots 4-6, dot 1, dots 2-3-4] Greek letter sigma (uppercase) (Σ)

New BANA Terms

- **Hollow dot:** The hollow dot has been used before within the degree symbol. In this chapter, it is used as an operation sign representing the composition of two functions.

Key Points

- A number enclosed within vertical bars is used to represent **absolute value**. The numeric indicator is not used when placing a number between two vertical bars.
- The **infinity symbol** is used to represent that a value continues forever in the positive direction. For negative infinity, the value continues forever in the negative direction.
- When writing **function notation,** parentheses are used.
- When writing **interval notation,** parentheses and/or brackets are used with a comma between the beginning of the interval and the end of the interval. A parenthesis means the value is not included in the interval and a bracket means the value is included in the interval.

- The **hollow dot** is used for degrees and also for composition of functions.
- In addition to the grouping symbols of parentheses and brackets, **angle brackets** are a type of grouping symbol most commonly used in math related to vectors.
- The **vertical bar** can be used in different ways including representing "such that" in set notation, the "norm of" in vectors, or the "determinant of" in matrices.
- Grouping symbols such as parentheses, brackets, braces, and vertical bars can be enlarged and are commonly used in matrices, determinants of matrices, and systems of equations.
- Problems involving **enlarged grouping symbols**
    - Are brailled as spatial arrangements, so leave a blank line above and below the arrangement.
    - Must have at least one enlarged grouping symbol on each line of the arrangement.
    - Must have the enlarged grouping symbols lined up vertically.
    - Sometimes have problem identifiers, parts of an expression, punctuation, etc. connected to the arrangement. These must be placed on the top line of the arrangement in braille regardless of its location in print.
- There are multiple symbols used in set theory including the **empty set symbol** or **null set**, the **union** symbol, the **intersection** symbol, the "**is a subset of"** symbol, and the "**is an element of"** symbol.
- The **union** symbol and the **intersection** symbol are signs of operation so there is no space on either side of the symbol.
- The "**is a subset of**" symbol and the "**is an element of**" symbol are signs of comparison so a space must be left on either side of these symbols.
- When an exponent (**superscript**) is not followed by a space or not at the end of a line, a **baseline indictor** is used to return to baseline after writing the superscript.
- When a **superscript indicator** or **subscript indicator** is used, and the superscript or subscript is immediately followed by a mathematical symbol, such as a sign of operation, grouping symbol, termination indicator, or fraction line, the **baseline indicator** is used to end the superscript or subscript and return to baseline.
- No baseline indicator is needed if a space or other level indicator follows a superscript or subscript.
- Do not use a **subscript indicator** or a **baseline indicator** when a variable has a numeric subscript.

- The **index of a radical indicator** is used to show a radical with an index such as a cube root, fourth root, etc.
- Common **functions** include sin, cos, tan, log, and ln. Regardless of whether there is a space in print between the function name and what comes after it, there is always a space in braille between the function name and what comes after it. Do not use an English letter indicator in a function.
- Common **Greek letters** include alpha, beta, and theta which are lowercase and delta and sigma which are uppercase. There are many other Greek symbols used in mathematics. A complete list of Greek symbols can be found in the appendices.

Introduction

In Algebra 1 and Algebra 2 students learn about expressions, equations, inequalities, tables, and graphs. In these classes, students learn to work with different types of functions such as linear, quadratic, polynomial, absolute value, piecewise-defined, exponential, logarithmic, rational, radical, and trigonometric functions. These functions are used to represent relationships between quantities.

Absolute Value

As explained in Chapter 6, students are introduced to absolute value in sixth grade (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015). Absolute value is represented by a vertical bar on each side of a number or expression. It takes one cell to write each **vertical bar** in Nemeth Code and is written with dots 1-2-5-6.

$|$

⠳

The vertical bar for absolute value is considered a grouping symbol in Nemeth Code. A number, whether positive or negative, enclosed within vertical bars that are used for absolute value, does not need a numeric indicator (Rule II §11e(8)).

$|5-8|=3$

⠳⠢⠤⠦⠳ ⠨⠅ ⠼⠒

> Read: The absolute value of five minus eight close absolute value equals three.

If two sets of vertical bars are placed next to each other, a multipurpose indicator (dot 5) must be placed between the two sets (Rule XXIII §177vii).

$|5||-3| = 15$

⠳⠢⠳⠐⠳⠤⠒⠳ ⠨⠅ ⠼⠂⠢

Additional Uses of the Vertical Bar

There are other uses of the vertical bar introduced in high school starting in Algebra 1, but the vertical bar is always written with dots 1-2-5-6 (Common Core State Standards Initiative, Appendix A, 2010).

As described in Chapter 6, braces are grouping symbols. One of the uses of braces in this chapter is to represent sets. When a set includes a list of numbers, no numeric indicators are used since it is an enclosed list (Rule 11, §10).

The left (open) brace (dots 4-6, dots 1-2-3-5-6) is written at the beginning of the set.

$\{$

⠨⠷

The right (close) brace (dots 4-6, dots 2-3-4-5-6) is written at the end of the set.

$\}$

⠨⠾

$\{10, 11, 12\}$

⠨⠷⠂⠴⠠ ⠂⠂⠠ ⠂⠆⠨⠾

Read: the set with elements ten, eleven, and twelve

One vertical bar can represent the words **such that**. In this case, there must be a space before and after the vertical bar in Nemeth Code (Rule XX, §145). This vertical bar is usually used within set notation to represent domain (possible values of x) or range (possible values of y). Domain and range written in set notation normally appear in a set of braces.

$\{x \mid x \neq 0\}$

⠨⠷⠭ ⠳ ⠭ ⠌⠨⠅ ⠼⠴⠨⠾

Read: The set of all x such that x is not equal to 0.

$\{y \mid -3 \leq y \leq 3\}$

⠨⠷⠽ ⠳ ⠤⠼⠒ ⠐⠅⠱ ⠽ ⠐⠅⠱ ⠼⠒⠨⠾

Read: The set of all y such that -3 is less than or equal to y is less than or equal to 3.

Often a **vector** is represented by a lowercase letter. A vector is an object that has both a magnitude and a direction. Picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow on one end indicating the direction. When the name of the vector is placed between two vertical bars, the expression means the **magnitude** of the vector.

$|u_1| = 5$

⠳⠥⠂⠳ ⠨⠅ ⠼⠢

Read: The magnitude of u sub 1 equals 5.

Sometimes the magnitude of a vector is written with two vertical bars before and after, but it still means magnitude whether there is a set of single vertical bars or a set of two vertical bars before and after the name of the vector.

$\|v\| = \sqrt{x^2 + y^2}$

⠳⠳⠧⠳⠳ ⠨⠅ ⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠻

Read: The magnitude of v equals the square root of x squared plus y squared end root.

A letter placed between vertical bars or double vertical bars can also mean the **norm of** something such as the norm of functions. In fact, a letter placed between vertical bars in mathematics has a lot of different contexts. However, in Nemeth Code vertical bars are always brailled the same way as previously described. The number of vertical bars in braille always follow the number of vertical bars used in print.

$\|g\|$

⠳⠳⠛⠳⠳

Read: the norm of g

$|x|$

⠳⠭⠳

Read: the norm of x

A capital letter is often used to represent the name of a matrix in algebra. When vertical bars are placed around this name, it means the **determinant of the matrix**. Later in the chapter, examples of matrices will be provided.

$|B| = -4$

⠳⠠⠃⠳ ⠨⠅ ⠤⠼⠲

Read: The determinant of matrix B equals negative four.

Example 8.1 contains three examples of absolute value and three examples of other uses for the vertical bar. Notice that in the third example the opening Nemeth Code indicator and Nemeth Code terminator are used in the word problem.

Example 8.1

$|-5| = |5| = 5$

⠳⠤⠢⠳ ⠨⠅ ⠳⠢⠳ ⠨⠅ ⠼⠢

Read: The absolute value of negative five equals the absolute value of five equals five.

$y = |8 - x|$

⠽ ⠨⠅ ⠳⠦⠤⠭⠳

Read: y equals open absolute value eight minus x close absolute value.

Solve for x if $|x-2| \leq 7$.

⠠⠎⠕⠇⠧⠑ ⠿ ⠰⠭ ⠊⠋ ⠸⠩ ⠳⠭⠤⠆⠳ ⠐⠅⠱ ⠼⠶ ⠸⠱⠲

Read: Solve for x if open absolute value x minus two close absolute value is less than or equal to seven.

$\{x \mid x > 1\}$

⠨⠷⠭ ⠳ ⠭ ⠨⠂ ⠼⠂⠨⠾

Read: The set of all x such that x is greater than one.

$|D| = 35$

⠳⠠⠙⠳ ⠨⠅ ⠼⠒⠢

Read: The determinant of D equals thirty-five.

$\|f - h\|$

⠳⠳⠋⠤⠓⠳⠳

Read: the norm of f minus h

Expanded Use of Grouping Symbols

As described in Chapter 6, parentheses and brackets are also grouping symbols. They are both frequently used in math expressions.

The left (open) parenthesis is written dots 1-2-3-5-6 and the right (close) parenthesis is written dots 2-3-4-5-6.

( )

⠷ ⠾

The left (open) bracket is written dot 4, dots 1-2-3-5-6 and the right (close) bracket is written dot 4, dots 2-3-4-5-6.

[ ]

⠈⠷ ⠈⠾

Here is an example to review the use of parentheses and brackets covered in previous chapters.

$2\left[3-\left(5+4\right)\right]$

⠼⠆⠈⠷⠒⠤⠷⠢⠬⠲⠾⠈⠾

> Read: two open bracket three minus open parenthesis five plus four close parenthesis close bracket

Parentheses are used in function notation, which students are first introduced to in Algebra 1 (Common Core State Standards Initiative, 2010; Maryland State Department of Education, 2015). A function, by definition, has no two points on the graph with the same x-coordinate.

Function notation is read "f of x" and is written using a single letter such as f followed by an x enclosed in parentheses. Notice that even though it is read "f of x," the word "of" is not written when using function notation. The function doesn't always have to be named f, but functions are commonly named using the letters f, g, or h.

$f\left(x\right)$

⠋⠷⠭⠾

Functions also have a domain and range. The domain refers to the set of inputs or x values and the range refers to the set of outputs or y values. The domain and range are often written in **interval notation**. Students are introduced to interval notation in Algebra 1 and Algebra 2 (Common Core State Standards Initiative, Appendix A, 2010).

Interval notation uses grouping symbols such as parentheses or brackets. The numbers at the beginning and the end of the interval are written inside the grouping symbols and are separated by a comma followed by a space.

When writing interval notation, a set of parentheses, a set of brackets, or one of each could be used. The parenthesis means that the number next to the parenthesis is not included in the interval. An open interval is written with both the opening and closing parentheses. The bracket means that the number next to the bracket is included in the interval. A closed interval is written with both the opening and closing brackets.

Another symbol often used in interval notation is the **infinity symbol**. It is used to represent that the interval continues forever. It takes two cells to write the infinity symbol in Nemeth Code. The infinity symbol is dot 6, followed by dots 1-2-3-4-5-6.

∞

⠠⠿

Example 8.2 contains five examples. The first two examples show functions, and the last three examples show interval notation. The last three examples use parentheses, brackets, or a combination of parentheses and brackets to write interval notation.

Example 8.2

$f(x) = -3(x+2) - 4$

⠋⠷⠭⠾ ⠨⠅ ⠤⠼⠒⠷⠭⠬⠆⠾⠤⠲

Read: f of x equals negative three open absolute value x plus two close absolute value minus four.

$g(x) = 7|x-5| + 1$

⠛⠷⠭⠾ ⠨⠅ ⠼⠶⠳⠭⠤⠢⠳⠬⠂

Read: g of x equals seven open absolute value x minus five close absolute value plus one.

$(-6,9)$

⠷⠤⠖⠠ ⠔⠾

Read: the open interval from -6 to 9

$[0,8]$

⠈⠷⠴⠠ ⠦⠈⠾

Read: the closed interval from 0 to 8

$(-\infty,12]$

⠷⠤⠠⠿⠠ ⠂⠆⠈⠾

Read: the interval from negative infinity to 12, including 12

Hollow Dot

The hollow dot is used for degrees and composite functions. Students are first introduced to the hollow dot used in degrees in elementary school and then composite functions in Algebra 1 or 2 (Common Core State Standards Initiative, Appendix A, 2010).

Remember that the degree symbol is three cells long and includes the superscript indicator (dots 4-5) followed by the hollow dot (dots 4-6, dots 1-6).

The **hollow dot** itself is two cells long and is written dots 4-6, dots 1-6.

$\circ$

⠨⠡

The hollow dot is also used when representing composite functions.

In algebra, a **composite function** is a function that depends on another function. It is created when one function is substituted into another function.

When reading a function, say "of" when you reach the hollow dot. To read $f \circ g$, say "f of g." This means that $g(x)$ would be substituted into $f(x)$ when creating the composite function.

$f \circ g$

⠋⠨⠡⠛

Angle Brackets

Angle brackets are first introduced when students start working with vectors in precalculus. Angle brackets are another type of grouping symbol (Common Core State Standards Initiative, Appendix A, 2010).

It takes three cells to write the angle brackets.

opening (left) angle bracket (dots 4-6, dots 4-6, dots 1-2-3-5-6)

closing (right) angle bracket (dots 4-6, dots 4-6, dots 2-3-4-5-6)

$\langle \; \rangle$

⠨⠨⠷ ⠨⠨⠾

The numeric indicator is not usually used when numbers are written inside of the angle brackets because the numbers are considered part of an enclosed list.

Example 8.3 contains five problems. The first example shows a hollow dot used to represent degrees. The second example shows the hollow dot used in composite functions. The third example shows that sometimes there can be a number instead of an x in the parentheses. The last two examples show the use of angle brackets.

Example 8.3

135°

⠼⠂⠒⠢⠘⠨⠡

Read: one hundred thirty-five degrees

$(h \circ g)(x) = h(g(x))$

⠷⠓⠨⠡⠛⠾⠷⠭⠾ ⠨⠅ ⠓⠷⠛⠷⠭⠾⠾

Read: h of g of x equals h of g of x.

$f(2) = -4(2) + 3$

⠋⠷⠆⠾ ⠨⠅ ⠤⠼⠲⠷⠆⠾⠬⠒

Read: f of two equals negative four open parenthesis two close parenthesis plus three.

$\langle -7, 60^\circ \rangle$

⠨⠨⠷⠤⠶⠠ ⠖⠴⠘⠨⠡⠐⠨⠨⠾

Read: open angle bracket negative seven comma sixty degrees close angle bracket

$$\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

⠨⠨⠷⠹⠂⠌⠆⠼⠠ ⠹⠜⠒⠻⠌⠆⠼⠨⠨⠾

Read: open angle bracket one-half comma square root of three end root over two close angle bracket

Practice 8.1

Interline the following problems that contain grouping symbols and hollow dots.

⠼⠂⠸⠲ ⠷⠛⠨⠡⠋⠾⠷⠖⠾ ⠨⠅ ⠳⠖⠳⠬⠆
⠼⠆⠸⠲ ⠈⠷⠤⠦⠠ ⠠⠿⠾
⠼⠒⠸⠲ ⠨⠷⠽ ⠳ ⠽ ⠐⠅ ⠼⠔⠨⠾
⠼⠲⠸⠲ ⠨⠨⠷⠶⠠ ⠒⠴⠘⠨⠡⠐⠨⠨⠾
⠼⠢⠸⠲ ⠳⠠⠍⠳ ⠨⠅ ⠤⠼⠂⠴

Use your braillewriter to write the problems that contain grouping symbols and hollow dots.

1. $\{x \mid x \geq 0\}$
2. $|C| = ad - bc$
3. $(h \circ f \circ g)(x) = \sqrt{x-2} + 1$
4. $(-2, 9]$
5. $\langle 3, 90^\circ \rangle$

Enlarged Grouping Symbols

Students are introduced to enlarged grouping symbols when they start working with systems of equations and piecewise-defined functions in Algebra 1 and matrices in precalculus (Common Core State Standards Initiative, Appendix A, 2010).

When enlarging a grouping symbol such as a parenthesis, bracket, brace, or vertical bar, a dot 6 is used before the last cell of the grouping symbol just like a dot 6 is used in front of a letter to capitalize it. Notice the placement of the dot 6 in the list below.

⠠⠷ [dot 6, dots 1-2-3-5-6] Enlarged opening (left) parenthesis (( )

⠠⠾ [dot 6, dots 2-3-4-5-6] Enlarged closing (right) parenthesis ( ))

⠈⠠⠷ [dot 4, dot 6, dots 1-2-3-5-6] Enlarged opening (left) bracket ([ )

⠈⠠⠾ [do 4, dot 6, dots 2-3-4-5-6] Enlarged closing (right) bracket ( ])

⠨⠠⠷ [dots 4-6, dot 6, dots 1-2-3-5-6] Enlarged opening (left) brace ({)

⠨⠠⠾ [dots 4-6, dot 6, dots 2-3-4-5-6] Enlarged closing (right) brace (})

⠠⠳ [dot 6, dots 1-2-5-6] Enlarged vertical bar (|)

The enlarged grouping symbol is used in braille when mathematical expressions are arranged on two or more lines in print and a sign of grouping is used to unify the lines (Rule XVIII §126). The enlarged grouping symbols are often used for piecewise-defined functions, systems of equations, matrices, and determinants.

Math Review:

- Piecewise-defined functions – a group of two or more functions, each having their own interval or portion of a single function. It is a function built from pieces of different functions over different intervals.
- Systems of equations – a group of two or more equations that are solved together to find the intersection of their graphs, if one exists.
- Matrices – rectangular array or table of numbers, symbols, or expressions, arranged in rows and columns
- Determinants – special numbers that can be calculated from square matrices (matrices with the same number of rows as columns)

Keep these points in mind when using enlarged grouping symbols:

- Arrangements with enlarged grouping symbols are considered spatial problems so leave a blank line above and below the arrangement (Rule XXV §185a).
- There must be at least one enlarged grouping symbol on each line of the arrangement (Rule XVIII §126).
- The enlarged grouping symbols must be lined up vertically (Rule XVIII §126).
- Follow print regarding the presence or absence of enlarged grouping symbols (Rule XVIII §126).
- Sometimes an arrangement using enlarged grouping symbols have problem identifiers, parts of an expression, punctuation, etc. connected to the arrangement. These must be placed on the top line of the arrangement in braille regardless of its location in print (Rule XXV §185b iv).

- For systems of equations, when elements of the system are vertically aligned in print, the alignment is retained in braille (Rule XVIII §128).
- For matrices and determinants:
    - Each entry must be moved as far left as possible in its column (Rule XXIV §183a).
    - One column of blank cells must be left between the columns of the arrangement (Rule XXIV §183a).
    - The numeric indicator must be used with numeric entries (Rule II §9c).

Keep these rules in mind when looking at the following example. In particular, notice there is a blank line after the directions and before the first spatial problem in the example below.

Example 8.4

Solve the following problems.

1. $y = \begin{cases} -x-2, x \le -2 \\ \sqrt{4-x^2}, -2 < x < 2 \\ x-2, x \ge 2 \end{cases}$

2. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 1 \\ 2 & 1 & 5 & 0 \end{array}\right]$

```
    ⠠⠎⠕⠇⠧⠑ ⠮ ⠋⠕⠇⠇⠪⠬ ⠏⠗⠕⠃⠇⠑⠍⠎⠲ ⠸⠩

⠼⠂⠸⠲ ⠽ ⠨⠅ ⠨⠠⠷⠤⠭⠤⠆⠠ ⠭ ⠐⠅⠱ ⠤⠼⠆
          ⠨⠠⠷⠜⠲⠤⠭⠘⠆⠐⠻⠠ ⠤⠼⠆ ⠐⠅ ⠭ ⠐⠅ ⠼⠆
          ⠨⠠⠷⠭⠤⠆⠠ ⠭ ⠨⠂⠱ ⠼⠆

⠼⠆⠸⠲ ⠈⠠⠷⠼⠂ ⠼⠂ ⠼⠂  ⠳⠼⠆⠈⠠⠾
     ⠈⠠⠷⠼⠴ ⠼⠂ ⠤⠼⠒ ⠳⠼⠂⠈⠠⠾
     ⠈⠠⠷⠼⠆ ⠼⠂ ⠼⠢  ⠳⠼⠴⠈⠠⠾

⠸⠱
```

In Problem 1 above, notice the problem number and the "y =" is lined up with the first line in braille even though is it lined up with the second line in print.

In Problem 2 above, which is called an **augmented matrix**, notice the following:

- The vertical bar is not considered enlarged because it is already in enlarged brackets
- The numeric indicators are used
- Each entry is moved as far left as possible in its column
- One column of blank cells is left between the columns of the arrangement.

Example 8.5 contains five examples of piecewise-defined functions, systems of equations, matrices, and determinants. The grouping symbols are lined up vertically in all problems.

In the first example, notice the variables a, b, and c are lined up in both print and braille.

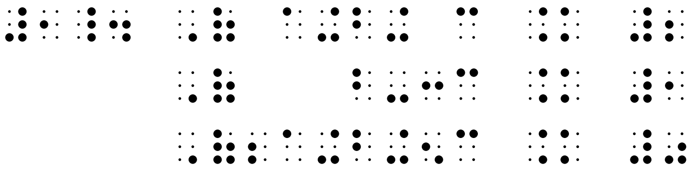
In the second example, notice the plus sign is on the first line in braille even though it is centered in print.

In the third example, notice the numeric indicators are used, each entry is moved as far left as possible in its column, and one column of blank cells is left between the columns of the arrangement. Also, notice that the words before the determinant and the question mark after the determinant are on the first line in braille even though they are centered in print.

All of the following examples could be read many different ways, so consult your math teacher to determine how they would like the problems read.

Example 8.5

1.
$$\left(\begin{array}{rrrl} a & +b & +c & =2 \\ & b & -3c & =1 \\ 2a & +b & +5c & =0 \end{array}\right.$$



2. $\begin{bmatrix} -7 & 5 \\ -10 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 6 & -2 \end{bmatrix}$

⠼⠆⠸⠲ ⠈⠠⠷⠤⠼⠶ ⠼⠢⠈⠠⠾⠬⠈⠠⠷⠤⠼⠲ ⠼⠴ ⠈⠠⠾
⠈⠠⠷⠤⠼⠂⠴ ⠼⠒⠈⠠⠾ ⠈⠠⠷⠼⠖ ⠤⠼⠆⠈⠠⠾

3. What is $\begin{vmatrix} -6 & 4 \\ 9 & -5 \end{vmatrix}$?

⠼⠒⠸⠲ ⠠⠱⠁⠞ ⠊⠎ ⠠⠳⠼⠤⠖ ⠼⠲ ⠠⠳⠸⠦
⠠⠳⠼⠔ ⠤⠼⠢⠠⠳

Non-Use of Enlarged Grouping Symbols

Do not use enlarged grouping symbols in the following situations (Rule XVIII §127):

- To cover a fraction

  $$\left(\frac{2x+4}{3}\right)\left(\frac{3x-6}{2}\right)$$

  ⠷⠹⠆⠭⠬⠲⠌⠒⠼⠾⠷⠹⠒⠭⠤⠖⠌⠆⠼⠾

  Read: open parenthesis open fraction two x plus four over three close fraction close parenthesis open parenthesis open fraction three x minus six over two close fraction close parenthesis

- To write a binomial coefficient

  The binomial coefficient uses regular parentheses. It also uses the directly under indicator (dots 1-4-6) between the top number and the bottom number. For more information, see Rule XIV §90.

  $$\binom{5}{4}$$

  ⠷⠢⠩⠲⠾

  Read: five choose four

- To write material occupying a large amount of vertical space such as a square root inside of another square root

  The first inner radical order-of-radical indicator (dots 4-6) is placed in front of the inner radical (dots 3-4-5) and the inner termination indicator (dots 1-2-4-5-6). For more information, see Rule XV §105.

  $\sqrt{\sqrt{125}}$

  ⠜⠨⠜⠂⠆⠢⠨⠻⠻

  Read: the square root of the square root of one hundred twenty-five

Practice 8.2

Interline the following problems that contain enlarged grouping symbols.

```
⠸⠩

⠼⠂⠸⠲ ⠷⠖⠩⠆⠾


⠼⠆⠸⠲ ⠛⠷⠭⠾ ⠨⠅ ⠈⠠⠷⠆⠭⠤⠂⠠ ⠠⠄⠿ ⠭ ⠐⠅ ⠼⠲
             ⠈⠠⠷⠢⠤⠭⠠ ⠠⠄⠿ ⠭ ⠨⠂⠱ ⠼⠲


⠼⠒⠸⠲ ⠨⠠⠷ ⠆⠭⠤⠒⠽ ⠨⠅ ⠼⠖
     ⠨⠠⠷⠤⠒⠭⠬⠲⠽ ⠨⠅ ⠼⠂⠆


⠼⠲⠸⠲ ⠠⠉ ⠨⠅ ⠈⠠⠷⠼⠶  ⠤⠼⠂ ⠼⠴ ⠈⠠⠾
           ⠈⠠⠷⠤⠼⠒ ⠼⠆  ⠤⠼⠔⠈⠠⠾


⠼⠢⠸⠲ ⠠⠳⠤⠼⠢  ⠼⠖ ⠠⠳
     ⠠⠳⠤⠼⠂⠴ ⠼⠂⠆⠠⠳


⠸⠱
```

Use your braillewriter to write the problems that contain enlarged grouping symbols.

1. $y = \begin{cases} -0.5x + 3, x \leq -2 \\ 7 + \frac{3}{4}x, x > -2 \end{cases}$

2. $\left(\begin{array}{r} -a + 5b = 10 \\ 7b = 21 \end{array} \right.$

3. $D = \begin{bmatrix} 0 & -2 \\ 6 & 10 \\ -8 & 1 \end{bmatrix}$

4. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

5. $\left(\frac{4}{x + y} \right) - \left(\frac{2}{x - y} \right)$

Set Theory

Students are introduced to set theory in geometry (Common Core State Standards Initiative, Appendix A, 2010).

One of the symbols used in set theory is the **empty set**, which is also referred to as the **null set**. An empty set is a set with no elements in it. There are two ways to represent the empty set. One way is to use a set of braces with a space between them.

$\{\ \}$

⠸⠷ ⠸⠾

Another way to write the empty set is to use the **empty set symbol**. It takes two cells to write the empty set symbol in Nemeth Code. The empty set symbol is written dots 4-5-6, dots 3-5-6.

$\varnothing$

⠸⠴

Two common signs of operation used in set theory are the union and intersection symbols. Since these symbols are considered signs of operation, there are no spaces on either side of the symbols.

It takes two cells to write the union symbol in Nemeth Code. The **union symbol** is written dots 4-6, dots 3-4-6.

∪

⠨⠬

Finding the union of two sets, is like uniting them or putting them together so that all of the combined elements are in one set.

> **Teaching Tip:** The plus sign is used in the union symbol. Point out to students that the union symbol is just like "adding" the two sets together. Another way to think about it is that it looks like a "U" or the print union symbol if you connect the dots.

It takes two cells to write the intersection symbol in Nemeth Code. The **intersection symbol** is written dots 4-6, dots 1-4-6.

∩

⠨⠩

Finding the intersection of two sets is like finding the intersection or overlap between the two sets. The intersection is the set of elements that are the same in both of the original sets.

> **Teaching Tip:** Since finding an intersection of two sets is the same as finding the elements that are the same or shared between two sets, point out to students that dots 1-4-6 in the intersection symbol is like the "sh" contraction used at the beginning of the word shared. Another way to think about it is that it looks like the print intersection symbol if you connect the dots.

Two common signs of comparison used in set theory are the "is a subset of" and "is an element of" symbols. These symbols are signs of comparison, so there is a space before and after the symbols.

It takes four cells to write the "**is a subset of"** symbol in Nemeth Code. It is written dots 4-5-6, dot 5, dots 4-6, dots 1-5-6.

$\subseteq$

⠸⠐⠅⠱

A subset is a smaller part of a bigger set.

Teaching Tip: Point out to students that the braille configuration for the "is a subset of" symbol includes a less than or equal to sign.

It takes two cells to write the **"is an element of"** symbol In Nemeth Code. It is written dot 5, dots 1-5.

$\in$

⠐⠑

An element is an object or member of a set. Capital letters can be used to represent the name of a particular set.

Teaching Tip: Point out to students that the letter "e" for "element" is in the braille configuration for the "is an element of" symbol.

Example 8.6 contains seven examples using set theory notation. In the first four examples, there is no space around the union or intersection symbols because they are signs of operation. In the last three examples, there is a space before and after the symbols for "is a subset of" and "is an element of" because they are signs of comparison.

Example 8.6

$A \cup B$

⠠⠁⠨⠬⠠⠃

Read: A union B

$\{1,3\} \cup \{2,4\} = \{1,2,3,4\}$

⠨⠷⠂⠠ ⠒⠨⠾⠨⠬⠨⠷⠆⠠ ⠲⠨⠾
⠨⠅ ⠨⠷⠂⠠ ⠆⠠ ⠒⠠ ⠲⠨⠾

Read: The set with elements one and three union the set with elements two and four equals the set with elements one, two, three, and four.

$\{1,3\} \cap \{2,3\} = \{3\}$

⠨⠷⠂⠠ ⠒⠨⠾⠨⠩⠨⠷⠆⠠ ⠒⠨⠾ ⠨⠅ ⠨⠷⠒⠨⠾

Read: The set with elements one and three intersection the set with elements two and three equals the set with element three.

$\{1,3\} \cap \{2,4\} = \varnothing$

⠨⠷⠂⠠ ⠒⠨⠾⠨⠩⠨⠷⠆⠠ ⠲⠨⠾ ⠨⠅ ⠸⠴

Read: The set with elements one and three intersection the set with elements two and four equals the empty set.

$\{2,4\} \subseteq \{1,2,3,4\}$

⠨⠷⠆⠠ ⠲⠨⠾ ⠸⠐⠅⠱ ⠨⠷⠂⠠ ⠆⠠ ⠒⠠ ⠲⠨⠾

Read: The set with elements two and four is a subset of the set with elements one, two, three, and four.

$\{\ \} \subseteq \{A,B,C,D\}$

⠨⠷ ⠨⠾ ⠸⠐⠅⠱ ⠨⠷⠠⠁⠠ ⠠⠃⠠ ⠠⠉⠠ ⠠⠙⠨⠾

Read: The empty set is a subset of the set with elements A, B, C, and D.

$4 \in \{2,4,6,8\}$

⠼⠲ ⠈⠑ ⠨⠷⠆⠠ ⠲⠠ ⠖⠠ ⠦⠨⠾

Read: Four is an element of the set with elements two, four, six, and eight.

Practice 8.3

Interline the following problems that contain symbols in set theory.

⠼⠂⠸⠲ ⠨⠷⠎⠠ ⠞⠨⠾ ⠸⠐⠅⠱ ⠨⠷⠎⠠ ⠑⠠ ⠞⠨⠾

⠼⠆⠸⠲ ⠠⠽⠨⠬⠸⠴ ⠨⠅ ⠠⠽

⠼⠒⠸⠲ ⠼⠂⠖ ⠈⠑ ⠨⠷⠂⠠ ⠲⠠ ⠔⠠ ⠂⠖⠠ ⠆⠢⠨⠾

⠼⠲⠼⠲ ⠨⠷⠏⠊⠛⠠ ⠉⠕⠺⠨⠾⠨⠩⠨⠷⠉⠁⠞⠠ ⠙⠕⠛⠨⠾

⠨⠅ ⠨⠷ ⠨⠾

⠼⠢⠼⠲ ⠠⠁⠨⠬⠠⠃ ⠨⠅ ⠨⠷⠂⠠ ⠒⠠ ⠢⠠ ⠂⠢⠨⠾

Use your braillewriter to write the following problems that contain symbols in set theory.

1. $\{10,15,20\}\cap\{10,20,30\}=\{10,20\}$
2. $D\subseteq A$
3. $\{a,b,c\}\cup\{x,y,z\}=\{a,b,c,x,y,z\}$
4. $a\in\{m,a,t,h\}$
5. $E\cap F=\varnothing$

Review of Superscripts and the Baseline Indicator

As introduced in Chapter 6, the **superscript indicator** (dots 4-5) is used to represent exponents, powers, or raised characters such as the degree sign or inverse. It is used after the base and before the actual raised character or exponent. There is not a corresponding sign in print for the superscript indicator. If a mathematical symbol, such as a sign of operation, grouping symbol, termination indicator, or fraction line, comes after the exponent, a **baseline indicator** (dot 5) is used after the exponent to indicate the exponent has ended and there is a return to baseline.

No baseline indicator is needed if a space or other level indicator follows a superscript or subscript. There is no print equivalent for the baseline indicator.

A common formula that uses superscript and baseline indicators is the Pythagorean Theorem for a right triangle with sides a, b, and c. The hypotenuse c is the longest side and opposite the right angle.

$$a^2+b^2=c^2$$

⠁⠘⠆⠐⠬⠃⠘⠆ ⠨⠅ ⠉⠘⠆

Read: a squared plus b squared equals c squared.

The superscript indicator is needed before each exponent. Following the "a squared," the baseline indicator is needed because the "a squared" is followed by the plus sign at baseline. No baseline indicator is needed following the "b squared" and "c squared" because each is followed by a space.

Expanded Use of Superscripts

In high school, students begin using superscripts and subscripts in more ways than explained in previous chapters (Common Core State Standards Initiative, Appendix A, 2010).

The superscript indicator is also needed to write the notation used for the inverse of a function. In braille, this is written f superscript negative 1 baseline indicator and then an x in parentheses. In this example, the negative 1 is not considered to be an exponent, but instead, refers to the inverse of the function.

$f^{-1}(x)$

⠋⠘⠤⠂⠐⠷⠭⠾

Read: f inverse of x

If an exponent is raised to another exponent, the superscript indicator is used twice instead of once before the second exponent (Rule XIII §74b).

The baseline indicator is not required because the double superscript indicator terminates the previous single superscript indicator. In other words, a level indicator terminates the effect of a previous level indicator and initiates the level implied by the new indicator (Rule XIII §79).

$e^{x^3} \cdot 3x^2$

⠑⠘⠭⠘⠘⠒⠐⠡⠒⠭⠘⠆

Read: e to the x cubed power times three x squared

Expanded Use of Subscripts

The **subscript indicator** (dots 5-6) is used to represent a lowered letter or number in all other cases. There is no print equivalent for the subscript indicator.

⠰

As described in Chapter 6, when subscripts are numbers associated with their respective letter or variable, no subscript indicator or corresponding baseline indicator is needed. The numeric subscript simply follows the variable with no space or any special symbol, and the baseline is maintained.

$$y - y_1 = m(x - x_1)$$

⠽⠤⠽⠂ ⠨⠅ ⠍⠷⠭⠤⠭⠂⠾

Read: y minus y sub one equals m open parenthesis x minus x sub 1.

Many chemistry formulas contain subscripts. The numbers appearing as subscripts in the chemical formula indicate the number of atoms of the element immediately before the subscript. If no subscript appears, one atom of that element is present. Additional examples for chemistry and physics can be found in the appendices.

$$C_6H_{12}O_6$$

⠠⠉⠖⠠⠓⠂⠆⠠⠕⠖

Read: C sub six H sub twelve O sub 6 or "fructose"

Similarly, when a numeric subscript follows a function such as log, no subscript indicator is used and a space is placed after the subscript (Rule XIII §77). There will be more information on these types of functions later in the chapter.

$$\log_3 x$$

⠇⠕⠛⠒ ⠭

Read: log base three of x

The next example requires the use of subscript indicators.

$$a_n = a_{n-1} + 5$$

⠁⠰⠝ ⠨⠅ ⠁⠰⠝⠤⠂⠐⠬⠢

Read: a sub n equals a sub n minus one baseline plus five.

Example 8.7 contains four examples using superscripts and subscripts.

In the first example, notice the subscript indicator is needed for the first subscript because the subscript is a letter. Also, the baseline is needed after the exponent because there is a parenthesis instead of a space after the exponent.

In the second example, notice the subscript indicator is not needed because the numeric subscript is associated with a letter.

In the third example, notice the subscript indicator is needed because the subscript is after a parenthesis instead of letter.

In the fourth example, notice the subscript indicator is not needed because the numeric subscript is associated with a function. Also, there is a space after the subscript because it is part of a function.

Example 8.7

$$S_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

⠠⠎⠰⠝ ⠨⠅ ⠹⠁⠂⠷⠂⠤⠗⠘⠝⠐⠾⠌⠂⠤⠗⠼

Read: S sub n equals open fraction a sub one open parenthesis one minus r to the nth power close parenthesis over one minus r close fraction.

$$\frac{d_1}{v_1} + \frac{d_2}{v_2} = t$$

⠹⠙⠂⠌⠧⠂⠼⠬⠹⠙⠆⠌⠧⠆⠼ ⠨⠅ ⠞

Read: Open fraction d sub one over v sub one close fraction plus open fraction d sub two over v sub two close fraction equals t.

$$Al\left(OH\right)_3$$

⠠⠁⠇⠷⠠⠕⠠⠓⠾⠰⠒

Read: Al open parenthesis OH close parenthesis sub three or "aluminum hydroxide"

$$\log_2 8 = 3$$

⠇⠕⠛⠆ ⠼⠦ ⠨⠅ ⠼⠒

Read: The log base two of eight equals three.

Expanded Use of Radicals

Students are introduced to radicals with an index in eighth grade (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015).

Before getting into the notation for radicals in Nemeth Code, let's review some terminology. The **radical expression** read as “the nth root of x” ($\sqrt[n]{x}$) has three major components:

1. the **radical symbol** (it looks like a check mark in print) $\sqrt{\ }$
2. the **index** (the small n tucked outside the radical symbol in print)
3. the **radicand** (x, the quantity written beneath the horizontal bar of the radical symbol in print).

An index of 5 means the fifth root. An index of 3 means the cube root. As mentioned in Chapter 6, an index of 2 is the square root, and the 2 is usually not written in print or braille.

When writing a radical with an index in Nemeth Code, follow these five simple steps.

1. The index-of-radical indicator (dots 1-2-6) ⠣
2. The index of the radical
3. The radical symbol (dots 3-4-5) ⠜
4. The radicand, value inside/under a radical symbol, which you want to find the root of
5. The termination indicator (dots 1-2-4-5-6) ⠻

No numeric indicator is used on the index or the radicand even if it is numeric.

The following steps outline how to write the cube root of 27 in Nemeth Code:

1. Index-of-radical indicator (dots 1-2-6) ⠣
2. Three (dots 2-5) ⠒
3. Radical symbol (dots 3-4-5) ⠜
4. Twenty-seven (dots 2-3, dots 2-3-5-6) ⠆⠶
5. Termination indicator (dots 1-2-4-5-6) ⠻

$\sqrt[3]{27}$

⠣⠒⠜⠆⠶⠻

The next example has an indexed radical and an exponent.

$\sqrt[6]{x^2} - 7$

⠣⠖⠜⠭⠘⠆⠐⠻⠤⠶

Read: The sixth root of x squared end root minus seven

Notice the words "end root" are used so that the reader knows the radical ends before the minus sign. Also, notice the baseline indicator is used after the exponent because the exponent was followed by the termination indicator instead of a space.

Example 8.8 has four examples of radicals with an index. Notice that in the last example an opening Nemeth Code indicator and Nemeth Code terminator are used in the word problem.

Example 8.8

$\sqrt[5]{\frac{1}{32}}$

⠣⠢⠜⠹⠂⠌⠒⠆⠼⠻

Read: The fifth root of open fraction one over thirty-two close fraction

$\sqrt[4]{y - 3}$

⠣⠲⠜⠽⠤⠒⠻

Read: The fourth root of y minus three end root

$\sqrt[6]{(x - 2)^6}$

⠣⠖⠜⠷⠭⠤⠆⠾⠘⠖⠐⠻

Read: The sixth root of open parenthesis x minus two close parenthesis to the sixth power end root

What is the $\sqrt[3]{0.125}$?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠸⠩ ⠣⠒⠜⠴⠨⠂⠆⠢⠻ ⠸⠱⠦

Read: What is the cube root of one hundred twenty-five thousandths?

Practice 8.4

Interline the following problems that contain superscripts, subscripts, and radicals.

⠼⠂⠸⠲ ⠼⠢⠘⠆⠐⠤⠣⠒⠜⠹⠂⠌⠦⠼⠻

⠼⠆⠸⠲ ⠇⠝ ⠑⠘⠽⠘⠘⠒ ⠨⠅ ⠽⠘⠒

⠼⠒⠸⠲ ⠋⠘⠤⠂⠐⠷⠭⠾ ⠨⠅ ⠣⠲⠜⠭⠤⠢⠻

⠼⠲⠸⠲ ⠷⠲⠤⠭⠘⠒⠐⠾⠘⠆ ⠨⠅ ⠼⠂⠖⠤⠦⠭⠘⠒⠐⠬⠭⠘⠖

⠼⠢⠸⠲ ⠁⠰⠝ ⠨⠅ ⠼⠆⠁⠰⠝⠤⠂⠐⠬⠶

Use your braillewriter to write the following problems that contain superscripts, subscripts, and radicals.

1. $f^{-1}(x) = \sqrt[4]{x} + 7$
2. $\log 10^{x^3} = x^3$
3. $a_c = \dfrac{v^2}{r}$
4. $\sqrt[4]{81x^8} = 3x^2$
5. H_2SO_4

Functions

Students are introduced to different type of functions in Algebra 1 and Algebra 2 (Common Core State Standards Initiative, Appendix A, 2010).

Common functions include:

- log (abbreviation for the common logarithm or log base 10)
- ln (abbreviation for the natural logarithm or log base e)
- cos (abbreviation for cosine)
- sin (abbreviation for sine)
- tan (abbreviation for tangent)

Keep these points in mind when using functions:

- The abbreviations are used in braille just like they are in print (Rule XVII).
- No contractions are used in the function names (Rule IX §55b).
- Whether or not there is a space after the function name in print, there is always a space after a function name in braille (Rule XVII §119a).
- If there is a single letter inside of a function, the English letter indicator is not used in Nemeth Code (Rule IV §27a).
- If a function has a numeric subscript, the subscript indicator is not used (Rule XIII §77).

Example 8.9 has five examples of functions.

In the first three examples, there is not a space in print after sin, cos, or tan, but there is a space in braille.

In the fourth example, no subscript indicator is used because the subscript on the function is numerical, and the space is after the numerical subscript.

In the fifth example, a subscript indicator is used because the subscript on the function is not numerical. In the last example, ln is used along with the letter e. These are commonly used together.

In the examples, there is no space when there is a function name following a letter, number, fraction indicators, or sign of operation (Rule XVII §119c).

Example 8.9

$\cos 135^\circ$

⠉⠕⠎ ⠼⠂⠒⠢⠘⠨⠡

Read: the cosine of one hundred thirty-five degrees

$\tan(-B) = -\tan B$

⠞⠁⠝ ⠷⠤⠠⠃⠾ ⠨⠅ ⠤⠞⠁⠝ ⠠⠃

Read: The tangent of open parenthesis negative B close parenthesis equals the negative tangent of B.

$\sin 2A = 2\sin A \cos A$

⠎⠊⠝ ⠼⠆⠠⠁ ⠨⠅ ⠼⠆⠎⠊⠝ ⠠⠁⠉⠕⠎ ⠠⠁

Read: The sine of two A equals two sine of A cosine of A.

$\log_3 81 = 4$

⠇⠕⠛⠒ ⠼⠦⠂ ⠨⠅ ⠼⠲

Read: The log base three of eighty-one equals four.

$\log_b a = \frac{\ln a}{\ln b}$

⠇⠕⠛⠰⠃ ⠁ ⠨⠅ ⠹⠇⠝ ⠁⠌⠇⠝ ⠃⠼

Read: The log base b of a equals open fraction the natural log of a over the natural log of b close fraction.

Greek Letters

In Chapter 7 you learned the Greek letter **pi**, which is written with the Greek letter indicator (dots 4-6), followed by the letter “p” (dots 1-2-3-4). Pi is a lowercase Greek letter often used when working with circles and trigonometry.

π

⠨⠏

Students are introduced to other uses for Greek letters in Algebra 1, geometry, Algebra 2, and beyond (Common Core State Standards Initiative, 2010; Maryland Department of Education, 2015). Both uppercase and lowercase Greek letters are often used in math.

When a Greek letter is capitalized, the order for writing the Greek letter is Greek letter indicator (dots 4-6), capital indicator (dot 6), letter. Common Greek letters include:

Names of angles

α alpha (lowercase)

⠨⠁

β beta (lowercase)

⠨⠃

θ theta (lowercase)

⠨⠹

To represent "change of"

Δ delta (uppercase)

⠨⠠⠙

To represent summation

Σ sigma (uppercase)

⠨⠠⠎

Greek letters are often used with trig functions such as sin, cos, and tan.

A complete list of Greek symbols can be found in the appendices.

Example 8.10 contains five examples with Greek letters.

Example 8.10

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

⠞⠁⠝ ⠨⠹ ⠨⠅ ⠹⠎⠊⠝ ⠨⠹⠌⠉⠕⠎ ⠨⠹⠼

> Read: The tangent of theta equals open fraction sine of theta over the cosine of theta close fraction.

The symbol Σ represents summation.

⠠⠮ ⠎⠽⠍⠃⠕⠇ ⠸⠩ ⠨⠠⠎ ⠸⠱ ⠗⠑⠏⠗⠑⠎⠢⠞⠎
⠎⠥⠍⠍⠁⠰⠝⠲

$$\sin^2\beta + \cos^2\beta = 1$$

⠎⠊⠝⠘⠆ ⠨⠃⠬⠉⠕⠎⠘⠆ ⠨⠃ ⠨⠅ ⠼⠂

> Read: Sine squared beta plus cosine squared beta equals one.

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$$

⠎⠊⠝ ⠷⠹⠨⠏⠌⠆⠼⠤⠨⠁⠾ ⠨⠅ ⠉⠕⠎ ⠨⠁

Read: The sine of open parenthesis open fraction pi over two close fraction minus alpha close parenthesis equals cosine of alpha.

$$E=-\frac{\Delta V}{\Delta s}$$

⠠⠑ ⠨⠅ ⠤⠹⠨⠠⠙⠠⠧⠌⠨⠠⠙⠎⠼

Read: E equals negative open fraction delta V over delta s close fraction.

Review of the Five-Step Rule from Chapter 7

When transcribing most modified expressions in Nemeth Code, the **five-step rule** is used (Rule XIV, §86). It is called the five-step rule because there are five steps that must be used in the same order each time an expression is modified using specified symbols (Rule XIV, §86). There is no print equivalent for the four modification indicators (multipurpose indicator, directly over indicator, directly under indicator, and termination indicator) used with the five-step rule.

Five-Step Rule

Step 1: Multipurpose indicator (dot 5) ⠐

Step 2: Expression being modified

Step 3: Directly over indicator (dots 1-2-6) ⠣

or **Directly under indicator** (dots 1-4-6) ⠩

Step 4: Modifier

Step 5: Termination indicator (dots 1-2-4-5-6) ⠻

Sigma Notation or Summation Notation

The Greek letter sigma is used in sigma notation. The symbol Σ represents summation and is used to represent the sum or total. Sigma notation includes symbols both above and below sigma so the "Five-Step Rule" needs to be used. Steps 3 and 4 will be repeated, once for what is below sigma and then again for what is above sigma.

When brailling an expression in sigma notation, here are the steps you will use. Notice the repetition of steps 3 and 4.

Step 1: Multipurpose indicator (dot 5) ⠐

Step 2: Expression being modified

sigma (uppercase) (dots 4-6, dot 6, dots 2-3-4) ⠨⠠⠎

Step 3: Directly under indicator (dots 1-4-6) ⠩

Step 4: Modifier (what is under sigma)

Step 3: Directly over indicator (dots 1-2-6) ⠣

Step 4: Modifier (what is over sigma)

Step 5: Termination indicator (dots 1-2-4-5-6) ⠻

$$\sum_{k=3}^{7} k^2$$

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Read: The sum from k=3 to 7 of k squared

When computing this expression, the student would square each of the numbers from 3 to 7, and then find the sum or total of those answers.

In other words, the expression written out in expanded form would be:

1. $\sum_{k=3}^{7} k^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2$

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Example 8.11 contains two more examples of sigma notation.

Example 8.11

$$\sum_{j=0}^{\infty}\left(\frac{1}{4}\right)^{j}$$

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Read: The sum from j=0 to infinity of one-fourth to the j power

$$\sum_{i=2}^{10} 5i - 3$$

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Read: The sum from i=2 to 10 of 5i minus 3

Practice 8.5

Interline the following problems that contain functions, Greek letters, and sigma notation.

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Use your braillewriter to write the following problems that contain functions, Greek letters, and sigma notation.

1. $R = R_0\left(1 + \alpha\Delta T\right)$
2. $\log_4 7 = \dfrac{\ln 7}{\ln 4}$
3. $\tan\theta = \dfrac{y}{x}$
4. $\sum_{k=0}^{\infty} 3\left(\dfrac{1}{5}\right)^{k}$
5. $\beta = 225^\circ$

Chapter Summary

Vertical Bar

- Positive or negative numbers enclosed in vertical bars are used to represent absolute value.
- Do not use a numeric indicator within the vertical bars when writing an absolute value.
- Vertical bars can be enlarged and are used for determinants in matrices.
- When a capital letter is placed between vertical bars it can represent the determinant of a matrix. An English letter indicator is not used with a letter placed between vertical bars.
- A single vertical bar can represent "such as" in set notation.
- A variable (letter) placed between one or two sets of vertical bars can represent the "magnitude of" or the "norm of."

Grouping Symbols

- Parentheses are used in function notation.
- The function f(x) is read "f of x."
- Brackets and parentheses are used to show interval notation. A bracket means the end value is included and a parenthesis means the end value is not included in the interval.
- Angle brackets are used to represent the component form when working with vectors.
- The numeric indicator is not usually used with numbers inside of the angle brackets because the numbers are considered part of an enclosed list. An exception to this rule is numbers in angle brackets that have the degree symbol after them.

Infinity

- The infinity symbol is used to represent that an interval continues forever.
- The infinity symbol can also be negative which means that the interval continues forever in the negative direction.

Hollow Dot

- The hollow dot is a superscript when used as degrees, so it is written with the superscript indicator followed by the hollow dot for degrees.
- The hollow dot is also used to represent a composite function. When reading the hollow dot in this case, say “of” when you reach the hollow

dot. For instance, to read $f \circ g$, say “f of g.” This means that g(x) would be substituted into f(x) when creating the composite function.

Enlarged Grouping Symbols

- When enlarging a grouping symbol, a dot 6 is used before the last cell of the grouping symbol just like a dot 6 is used in front of a letter to capitalize it.
- The enlarged grouping symbol is used in braille when there is a grouping symbol around multiple lines in print.
- Arrangements with enlarged grouping symbols are considered spatial problems so leave a blank line above and below the arrangement.
- There must be at least one enlarged grouping symbol on each line of the arrangement.
- The grouping symbols must be lined up vertically.
- Sometimes an arrangement using enlarged grouping symbols have problem identifiers, parts of an expression, punctuation, etc. connected to the arrangement. These must be placed on the top line of the arrangement in braille regardless of its location in print.
- Follow print regarding the presence or absence of grouping symbols.
- Enlarged grouping symbols are often used for piecewise-defined functions, systems of equations, matrices, and determinants.
- For systems of equations, when elements of the system are vertically aligned in print, the alignment is retained in braille.
- For matrices and determinants:
    - Each entry must be moved as far left as possible in its column.
    - One column of blank cells must be left between the columns of the arrangement.
    - The numeric indicator must be used with numeric entries.
- Do not use enlarged grouping symbols in the following situations:
    - To cover a fraction
    - To write a binomial coefficient
    - To write material occupying a large amount of vertical space such as a square root inside of another square root

Set Theory

- An empty set, or null set, is a set with no elements in it. There are two ways to represent the empty set. One way is to use a set of braces with a space between them. The other is to use the empty set symbol.
- The union and intersection symbols are considered signs of operation, so there are no spaces on either side of the symbols.

- The "is a subset of" and "is an element of" symbols are considered signs of comparison, so there should be a space before and after the symbols.

Superscripts and Subscripts

- The superscript indicator is used to represent exponents, powers, or raised characters such as the degree sign or inverse. It is used after the base and before the actual raised character or exponent. There is not a corresponding sign in print for the superscript indicator.
- If a mathematical symbol, such as a sign of operation, grouping symbol, termination indicator, or fraction line, comes after the exponent, a baseline indicator is used after the exponent to indicate the exponent has ended.
- No baseline indicator is needed if a space or other level indicator follows a superscript or subscript.
- There is no print equivalent for the baseline indicator.
- If an exponent is raised to another exponent, the superscript indicator is used twice, instead of once, before the second exponent.
- A level indicator terminates the effect of a previous level indicator and initiates the level implied by the new indicator
- The subscript indicator is dots 5-6 and is used to represent a lowered letter or number. There is no print equivalent for the subscript indicator.
- When subscripts are numbers associated with their respective letter or variable, no subscript indicator or corresponding baseline indicator is needed. The numeric subscript simply follows the variable with no space or any special symbol, and the baseline is maintained.
- Many chemistry formulas contain subscripts. The numbers appearing as subscripts in the chemical formula indicate the number of atoms of the element immediately before the subscript. If no subscript appears, one atom of that element is present.

Radicals with and without an Index

- The radical symbol begins a square root, and the termination indicator ends the square root.
- The numeric indicator is not used between the radical symbol and the termination indicator.
- There is no print equivalent for the termination indicator.
- The square root is assumed to have an index of 2.
- The index-of-radical-indicator is used for radicals with an index other than 2.

- The index is framed between the index-of-radical indicator and the radical symbol.

Functions

- Common functions include log, ln, sin, cos, and tan.
- The 3-letter abbreviations are used in braille just like they are in print. No contractions are used in the function names.
- Whether or not there is a space after the function name in print, there is always a space after a function name in braille.
- If there is a single letter inside of a function, the English letter indicator is not used.

Greek letters

- The Greek letter indicator is used when writing Greek letters.
- When a Greek letter is capitalized, the order for writing the Greek letter is Greek letter indicator, capital indicator, letter.
- Common Greek letters include alpha, beta, theta, pi, delta, and sigma.
- Greek letters are often used with trig functions such as sin, cos, and tan.

Sigma Notation

- The Greek letter sigma is used in sigma notation.
- This notation is used to represent the sum or total.
- Sigma notation includes symbols both above and below sigma, so the "Five-Step Rule" must be used. Step 3 and 4 will be repeated, once for what is below sigma and then again for what is above sigma.

Chapter 8: Answer Key

Answer 8.1

1. $(g \circ f)(6) = |6| + 2$

2. $[-8, \infty)$

3. $\{y \mid y < 9\}$

4. $\langle 7, 30^\circ \rangle$

5. $|M| = -10$

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Answer 8.2

1. $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

2. $g(x) = \left[\begin{array}{l} 2x - 1, \text{for } x < 4 \\ 5 - x, \text{ for } x \geq 4 \end{array}\right.$

3. $\begin{cases} 2x - 3y = 6 \\ -3x + 4y = 12 \end{cases}$

4. $C = \begin{bmatrix} 7 & -1 & 0 \\ -3 & 8 & -9 \end{bmatrix}$

5. $\begin{vmatrix} -5 & 6 \\ -10 & 12 \end{vmatrix}$

Answer 8.2 (continued)

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Answer 8.3

1. $\{s,t\} \subseteq \{s,e,t\}$
2. $Y \cup \varnothing = Y$
3. $16 \in \{1,4,9,16,25\}$
4. $\{pig, cow\} \cap \{cat, dog\} = \{\ \}$
5. $A \cup B = \{1,3,5,15\}$

Answer 8.3 (continued)

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Answer 8.4

1. $5^2 - \sqrt[3]{\frac{1}{8}}$

2. $\ln e^{y^3} = y^3$

3. $f^{-1}(x) = \sqrt[4]{x-5}$

4. $\left(4 - x^3\right)^2 = 16 - 8x^3 + x^6$

5. $a_n = 2a_{n-1} + 7$

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Answer 8.5

1. $\log_b xy = \log_b x + \log_b y$

2. $\sum_{n=5}^{8} 4n+2$

3. $\cos(2\beta) = 2\cos^2\beta - 1$

4. $\alpha = -\frac{\pi}{2}$

5. $I = \frac{\Delta Q}{\Delta t}$

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